Sequential Loop Closing Identification of Hammerstein Models for Multiple-Input Multiple-Output Processes

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Abstract: A lot of industrial chemical processes contain certain input nonlinearities even though they are controlled by several linear controllers. Here we investigate a sequential loop closing identification method for MIMO Hammerstein nonlinear processes with diagonal nonlinearities. The proposed method separates the identification of the nonlinear static function from that of the linear subsystem by using a relay feedback test and a triangular type signal test. From 2^n activations for n-input MIMO nonlinear processes, we sequentially identify the whole range of the nonlinear static function as well as the transfer function matrix of the linear subsystem.

Keywords: biased relay, diagonal nonlinearity, nonlinear static function, hammerstein nonlinear process, sequential loop closing

1. Introduction

Designing control systems of multivariable processes is still a challenge to the control engineers due to their complex interactive natures. Many control schemes for interacting multivariable processes are available. Though full multivariable control systems such as a model predictive control system provide good control performances, multi-loop control systems are often used in the chemical processes because of their simplicity and robustness.

The frequency response method can be used to identify parameters of several linear industrial processes. Astrom and Hagglund[1] identified ultimate information from a relay feedback test to tune the PID controller automatically. Also, Shen et al.[2] suggested an asymmetric biased relay feedback method to estimate steady state gain and ultimate information. Their idea has been applied in many areas. The sequential loop closing (SLC) method is one of the well-known methods to tune multi-loop control systems for the Multiple-input Multiple-output (MIMO) processes. In the SLC method, each controller is designed sequentially from the dynamics of each pair of inputs and outputs while the former controllers are closed. The dynamics of the latter pair of inputs and outputs can be changed considerably when the former loops are closed. Hence, the control performance can be very sensitive to which loop is tuned first and how it is designed. To avoid this drawback of SLC method, Shen and Yu[3] suggested several iterations with a conservative Zigler-Nichols tuning method.

Field tests for tuning of control systems are the most time-consuming step. Hence, it is desirable to reduce the number of field tests. For this, Choi et al.[4] proposed the sequential loop closing identification method to identify the whole transfer function. Also, Koo et al.[5] suggest a sequential loop closing identification method for multi-loop control system using an asymmetric biased relay feedback test. The identified models can be used to correct pairings for multi-loop control systems, and to improve the tuning performance without field iterations.

Linear models have inherent limitations in describing the nonlinear dynamics of industrial chemical processes. For better model performance, many authors have exerted much of their efforts in developing nonlinear system identification methods. One of the nonlinear black box models is the particular type model composed of linear dynamic subsystem and memoryless nonlinear static function such as Wiener, Hammerstein, Hammerstein-Wiener, and so on[6].


Among various type nonlinear models, we consider MIMO Hammerstein nonlinear processes with diagonal nonlinearity. If a nonlinear static element precedes a linear dynamic system, the model is called a Hammerstein-type nonlinear process as shown in Figure 1. In this research, we propose a sequential loop closing identification method for MIMO Hammerstein nonlinear processes. The proposed method separates the identification of the nonlinear static function from that of the linear subsystem by using a biased relay feedback method. By 2^n activations of the processes, we sequentially identify the whole range of the nonlinear static function as well as the transfer function matrix of the linear subsystem, separately.

\[ u(t) \rightarrow F(u(t)) \rightarrow v(t) \rightarrow \text{Linear Dynamic Subsystem} \rightarrow y(t) \]

Fig. 1. MIMO Hammerstein nonlinear process.
II. MIMO (Multiple-Input Multiple-Output) Hammerstein Nonlinear Processes

Consider a following block-oriented MIMO Hammerstein nonlinear process as shown in Figure 1. It can be represented by

\[ u(t) = \begin{bmatrix} u_1(t) & u_2(t) & \cdots & u_n(t) \end{bmatrix}^T \]

(1)

\[ y(t) = F(u(t)) = \begin{bmatrix} f_1(u_1(t)) & f_2(u_2(t)) & \cdots & f_n(u_n(t)) \end{bmatrix}^T \]

where

\[ G(s) = \begin{bmatrix} g_{11}(s) & \cdots & g_{1n}(s) \\
\vdots & \ddots & \vdots \\
g_{n1}(s) & \cdots & g_{nn}(s) \end{bmatrix} \]

(2)

\[ g_{ij}(s) = \frac{Y_i(s)}{V_j(s)} = \frac{K_{ij} \exp(-\theta_{ij}s)}{(\tau_{ij}s + 1)} \]

Where, \( u(t), y(t) \) and \( v(t) \) denote the process input, the output of the nonlinear function and the process output, respectively. \( G(s) \) is a stable transfer function matrix with several linear dynamic subsystems. Also \( f(*) \) means a nonlinear static function to be continuous and monotone. The main issue of this research is to identify each elements of the nonlinear static function matrix and to identify each transfer function of the linear dynamic subsystem matrix.

III. Identification of SIMO (Single-Input Multiple-Output) Hammerstein Nonlinear Processes

To identify sequentially an \( n \times n \) Hammerstein nonlinear process, an identification of a SIMO Hammerstein nonlinear process is required in Figure 2. Here, we suggest a two-step approach for SIMO process. In the first test, a biased relay feedback method is applied to activate the process to estimate the frequency responses of the linear dynamic subsystem. Subsequently, a triangular test input activates the process to identify the nonlinear static function.

1. Biased relay feedback and Frequency Responses of Linear Dynamic Subsystem

Consider the biased relay feedback experiment of Figure 2(a). We need to notice that the output of the relay is a binary signal and so is the output of the nonlinear static function of which period is still same while the mean value and the oscillation magnitude are changed. So, the following linear equation is valid between the two binary signals.

\[ v_{r1}(t) = \alpha u_{r1}(t) + \beta \]

(3)

where, \( \alpha \) and \( \beta \) are constants. \( u_{r1}(t) \) and \( v_{r1}(t) \) represent the relay output and the corresponding output of the nonlinear static function, respectively. Keeping this relationship in mind, let's consider the following transfer function for nonzero frequencies.

\[ g_{ii}(k\omega_o) = \frac{1}{2\pi} \int_{+P}^{+P} \frac{s-i\omega_o}{s+1} \exp(-jko,t) \, dt \]

(4)

Where, \( \omega_o = 2\pi P \) and \( P \) represent the relay frequency and the relay period, respectively. \( y_{r1}(t) \) and \( g_{ii}(k\omega_o) \) for \( i = 1, 2, 3, \ldots, n \) are the \( i \)-th activated process output by the relay feedback and the frequency responses of the process, respectively. It should be noted that the bias term of \( \beta \) does not affect the estimates of \( g_{ii}(k\omega_o) \). Therefore, the transfer function of (4) can be rewritten with respect to the transfer function of the linear dynamic subsystem as follows.

\[ g_{ii}(k\omega_o) = \alpha \frac{1}{2\pi} \int_{+P}^{+P} \frac{s-i\omega_o}{s+i} \exp(-jko,t) \, dt = \alpha g_{ii}(k\omega_o) \]

(5)

for \( k = 1, 2, 3, \ldots \)

where, \( g_{ii}(s) \) is the transfer function of the linear dynamic subsystem in Figure 2. Finally, we can assume that the transfer function of the linear dynamic subsystem is \( g_{ii}(s) \) without loss of generality because we have one degree of freedom to replace the static nonlinearity \( f(*) \) by \( f(*)-f(*)/u \) not to change the process input-output relationship. Now, it is clear that frequency responses of the linear dynamic subsystem at the frequencies \( jk\omega_o \), \( k = 1, 2, 3, \ldots \) can be estimated by (4) from the relay output and the corresponding process output.

2. Identification of Nonlinear Static Function and Linear Dynamic Subsystem

We estimated all frequency responses of the linear dynamic subsystem corresponding to the multiples of the relay frequency. In this section, we estimate the whole activated region of the nonlinear static function using a triangular periodic test signal of which period is the same as that of the relay output. Figure 2(b) shows the triangular test input activation. Here, the process output can be described as the following Fourier series expansion.

\[ y_{s1}(t) = A_0 + \sum_{k=1}^{\infty} \left( A_k \cos(k\omega_o t) + B_k \sin(k\omega_o t) \right) \]

(6)
\[ A_k = \frac{2}{\pi r} \int_{-\pi r}^{\pi r} y_{s1}(t) \cos \frac{2k\pi t}{\pi r} \, dt \]

\[ B_k = \frac{2}{\pi r} \int_{-\pi r}^{\pi r} y_{s1}(t) \sin \frac{2k\pi t}{\pi r} \, dt \]  

where, \( y_{s1}(t) \) denotes the \( i \)-th activated process output by the triangular signal.

To identify the nonlinear static function, we will reconstruct \( v_{s1}(t) \) from \( y_{s1}(t) \). Consider the following transfer function representation of the process.

\[ V_{s1}(s) = g_{i1}(s)^{-1} Y_{s1}(s) = \Psi_{i1}(s) Y_{s1}(s) \]  

where, \( g_{i1}(s)^{-1} = \Psi_{i1}(s) \) and \( \Psi_{i1}(s) \) are the output of the nonlinear static function and the first process output for the triangular-type signal. From (6) and (10), we have

\[ v_{s1}(t) = A_0 \Psi_{i1}(0) + z_1(t) \]

\[ z_1(t) = A_0 \left[ \sum_{k=1}^{\infty} B_k \Psi_{i1}(kJo_\tau) \right] \sin(ko_\tau t + z_1 Y_{s1}(kJo_\tau)) \]

\[ + \sum_{k=1}^{\infty} B_k \Psi_{i1}(nJo_\tau) \left[ \cos(no_\tau t + z_1 \Psi_{i1}(nJo_\tau)) \right] \]  

where, \( g_{i1}(j\omega_0) \) is estimated by (4). The coefficients of \( A_0 \) \( B_k \) are calculated from (8) and (9) with numerical integration of the process output. Then, we can obtain a data set of \( u_{d1}(t) \) versus \( z_1(t) \) from (12). Since \( u_0(t) \), \( d(t) \) and \( y(t) \) can be assumed to be deviation variables, we have \( f(t) = 0 \). Therefore, we can set \( A_0 \Psi_{i1}(0) \) in equation (11) be the value of \( z_1(t) \) corresponding to \( u_{d1}(t) = 0 \). The nonlinear static function is identified from a data set of the calculated \( v_{s1}(t) \) and \( u_{d1}(t) \). If an inverse polynomial model of the nonlinear function is needed, we can analytically solve the following optimization problem using the least squares method.

\[ \min_{\hat{g}_{i1}(s)} \sum_{i=1}^{N} \left( u_{d1}(t_i) - \hat{u}_{d1}(t_i) \right)^2 \]

subject to

\[ \hat{u}_{d1}(t) = \hat{g}_{i1} v_{i1}(t) + \hat{g}_{i2} v_{i2}(t) + \cdots + \hat{g}_{in} v_{iN}(t) \]  

After an identification of the nonlinear static function, we can obtain \( v_{i1}(t) \) from the obtained nonlinear function and \( u_{d1}(t) \). The first order plus time delay (FOPDT) model of \( g_{i1}(s) \) can be estimated as follows.

\[ \hat{g}_{i1}(s) = \frac{K_{p1i} \exp(-\theta_{p1i}(s))}{\tau_{p1i}(s+1)} \]  

where, \( K_{p1i} = g_{i1}(0) = \int_{-\pi r}^{\pi r} y_{s1}(t) \, dt / \int_{-\pi r}^{\pi r} v_{s1}(t) \, dt \),

\[ \tau_{p1i} = \sqrt{\left[ \frac{K_{p1i}}{g_{i1}(j\omega_0)} \right]^2 - 1} \]

\[ \theta_{p1i} = -\tan^{-1}\left( \frac{\tau_{p1i}(\omega_0) - \omega_\theta}{\omega_0} \right) \]

\[ \omega_\theta = \frac{\omega_\eta \omega_0}{\omega_0 + \omega_\eta} \]

\[ \omega_\eta = \omega_0 \exp(-\kappa T) \]

where, \( g_{i1}(j\omega_0) \) is a multi-valued function and hence the equation for the time delay has the term of \( \kappa P_{i1} \). When time delay is smaller than the oscillation period, \( \kappa \to 0 \). Otherwise, \( \kappa \) is an integer part of \( \theta_{p1i}(s) \). Experimentally, it can be found from the initial response of relay oscillation.

**IV. Sequential Loop Closing Identification of MIMO Hammerstein Nonlinear Processes**

In the sequential loop closing (SLC) method, since each controller is tuned sequentially by the ultimate information between the paired input and output, process model is not required. However, in the proposed identification method, we identify the transfer function matrix as well as the nonlinear static function. Each controller contains the inverse of the nonlinear static function and conventional PI controller tuned by the identified model. For \( n \times n \) Hammerstein nonlinear processes, only \( 2 \times n \) perturbations will be performed.

1. **First Activations and Feedback loop**

The first activations are put to the first input \( u_1(t) \) while other loops are open, as shown in figure 2. From these two activations, we can estimate \( f_1(t) \) and \( g_{i1}(s) \) for \( i = 1, 2, 3, \ldots, n \). Then, the first feedback controller including \( g_{i1}(s) \) of PI controller and \( f_1(t) \) can be designed as shown in Figure 3. Where, \( \eta(t) \) denotes the \( i \)-th set point and \( g_{i1}(s) \) is tuned by \( g_{i1}(s) \).

2. **(m+1)-th loop Activation (former \( m \)-loops are closed)**

It is very simple to expand the above SIMO identification method to \( n \times n \) processes. First, we identify \( g_{i1}(s) \) and \( f_1(t) \) from the first activations. Thereafter, the first loop is closed. Then, we assume that the first nonlinear function is completely removed. When the first loop is closed and \( \eta(t) \) and other inputs except \( u_1(t) \) are equal to zero, the MIMO Hammerstein nonlinear process can be treated as SIMO Hammerstein nonlinear process of which input is \( u_1(t) \). Then, we have a following SIMO process.

![Fig. 3. First feedback loop for MIMO process.](image-url)
\[ v_2(t) = f_2(u_2(t)) \]
\[ -u_1 \ y_2 \ y_3 \ \cdots \ y_n \]  
\[ = Q_2(s)v_2 \]  
\[ Q_2(s) = \begin{bmatrix} e_{c2}(s) & 0 & \cdots & 0 \\ 0 & I_{n-1} \end{bmatrix} + \begin{bmatrix} g_{1}(s) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ g_{n-1}(s) & 0 & \cdots & 0 \end{bmatrix}^{-1} G_2(s) \]  
\[ Q_{m+1}(s) = \begin{bmatrix} e_{c1}(s) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ g_{m}(s) & 0 & \cdots & 0 \end{bmatrix}^{-1} G_{m+1}(s) \]  
\[ G_{cm}(s) = \begin{bmatrix} e_{c1}(s) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_{cm}(s) \\ 0 & \cdots & 0 & I_{n-m} \end{bmatrix} \]

where, \( Q_{cm}(s) = [q_{cm1}(s) \ q_{cm2}(s) \ \cdots \ q_{cmn}(s)]^T \) and \( I_k \) denotes the \( k \times k \) unity matrix.

The second activations are put to the second input \( u_2(t) \) sequentially as shown in Figure 4. Then, the second loop will oscillate due to the activations. By analyzing oscillations in similar manner to the first activations, we can obtain the second nonlinear element \( f_2(s) \) and frequency responses of \( Q_2(s) \) at frequencies of \( 0, 2\pi P, 4\pi P, \ldots \), where \( P \) is the oscillation period. Since \( g_k(s) \) for \( k = 1, 2, 3, \ldots, n \) are already identified in the first activations, we can obtain frequency responses of \( g_k(s) \) from (17). Then FOPTD model parameters of \( g_k(s) \) can be estimated by (15). It is remarked that \( u_1(t) \) instead of \( y_1(t) \) is used to identify \( g_k(s) \). The reason is that, due to the integral action in the first feedback controller, we cannot obtain the steady state gain of \( g_k(s) \) from the oscillation of \( y_1(t) \). If the P controller, which doesn't include the integral action, is used as \( g_2(s) \), there is no such problem and \( y_1(t) \) can be used. Then, the second feedback controller can be designed from estimated \( f_2(s) \) and \( g_2(s) \).

Similarly, after former \( m \)-loops are closed, if former \( m \)-set points and latter inputs except \( u_{m+1}(t) \) are equal to zero, the overall SIMO Hammerstein nonlinear process is as follows.

\[ v_{m+1}(t) = f_{m+1}(u_{m+1}(t)) \]
\[ -u_1 \ \cdots \ -u_m \ y_{m+1} \ \cdots \ y_n \]  
\[ = Q_{m+1}(s)v_{m+1} \]  
\[ Q_{m+1}(s) = \begin{bmatrix} e_{c1}(s) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ g_{m}(s) & 0 & \cdots & 0 \end{bmatrix}^{-1} G_{m+1}(s) \]  
\[ G_{cm}(s) = \begin{bmatrix} e_{c1}(s) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_{cm}(s) \\ 0 & \cdots & 0 & I_{n-m} \end{bmatrix} \]

Note that when the loops are sequentially closed, each feedback controllers contain inverses of the nonlinear static functions. In this reason, we assume that the nonlinear elements of each loop perfectly eliminated. The transfer functions are identified column-wisely and the nonlinear elements are identified sequentially.

**V. Case Study**

Simulations are carried out to show that the proposed identification method can estimate the nonlinear static function matrix and the linear transfer function matrix.

1. 2x2 Hammerstein nonlinear process

Consider the Wood and Berry column with following input nonlinear elements.

\[ v(t) = \begin{bmatrix} v_1(t) \\ 6/(1+\exp(-u_2(t)))+3 \end{bmatrix} \]

\[ \begin{cases} -2 & \text{for } u_1(t) < -2 \\ u_1(t) & \text{for } -2 \leq u_1(t) \leq 2 \\ 2 & \text{for } u_1(t) \geq 2 \end{cases} \]

As shown in (20), the first nonlinear element is a saturation nonlinearity and the second nonlinear element contains exponential function. The biased relay is used to estimate frequency information of the first column linear subsystem. After the relay oscillation of the first loop as shown in Figure 5, a triangular type input with same period of relay feedback test is applied in the first loop. Using (11) and (12), we can get the data set of the first nonlinear function as shown in Figure 6. After the estimation of the nonlinear function, using (15), we can identify the transfer functions of the first column elements as follows.

\[ g_{11}(s) = 12.7991\exp(-1.005s)/(16.6996s + 1) \]
\[ g_{21}(s) = 6.6015\exp(-7.0049s)/(10.9019s + 1) \]

From the identified transfer function \( g_{11}(s) \), we can design the first loop controller by the SISO IMC-Pi method as \( g_{21}(s) = 0.1344 \) \((1+1/17.2021s)\) and close the first feedback loop. Under the first feedback loop closed, the second activations are put to \( u_2(t) \). Figure 7 shows the effect of the triangular type input for \( u_2(t) \). From these activated outputs, we can estimate the second nonlinear function and the second column elements of the transfer function matrix by using (17).
The obtained transfer function matrix is

\[
G(s) = \begin{bmatrix}
12.7991 \exp(-1.005s) & -23.1102 \exp(-3.0046s) \\
16.6996s + 1 & 20.9941s + 1 \\
6.6015\exp(-7.0049s) & -23.7447\exp(-3.0052s) \\
10.9019s + 1 & 14.4013s + 1
\end{bmatrix}
\]

and the estimated nonlinear function is described in figure 8. Note that the identified process gains of the second column elements are different from the Wood and Berry column. However the difference of the gain is compensated in the estimation of nonlinear static function, as shown in figure 8.

2. 3 × 3 Hammerstein nonlinear process

Consider the Ogunnaike and Ray system with following input nonlinearities

\[
v(t) = \begin{bmatrix} 6/(1 + \exp(-u_1(t))) + 3 & 0 & 0 \\ 0 & v_2(t) & 0 \\ 0 & 0 & v_3(t) \end{bmatrix}
\]

(21)

where, \( v_2(t) = \begin{cases} -2 & \text{for } u_2(t) < -2 \\ u_2(t) & \text{for } -2 \leq u_2(t) \leq 2 \text{ and } \\ 2 & \text{for } u_2(t) \leq 2 \end{cases} \)

\[
v_3(t) = \begin{cases} -\frac{\sqrt{u_3(t)}}{\sqrt{u_3(t)}} & \text{for } u_3(t) < 0 \\ \frac{\sqrt{u_3(t)}}{\sqrt{u_3(t)}} & \text{for } u_3(t) \geq 0 \end{cases}
\]

With the same approach in the example 1, the first activation is applied in the first input while other loops are open. Figure 9 shows the activated process output by relay feedback test for the first input. After an estimation of the transfer functions of the first column elements and the first nonlinear function, we tune the first loop controller. Sequentially, other loops are identified and closed.

Controllers designed by the SISO IMC-PI tuning method sequentially are

\[
\begin{align*}
g_{c1}(s) &= 1.0614(1 + 1/8.0025s) \\
g_{c2}(s) &= -0.2740(1 + 1/6.5040s)
\end{align*}
\]
VI. Conclusions

We suggested a method to identify a MIMO Hammerstein nonlinear process models while multiloop control systems are being tuned. We propose the series activation method that consist of a biased relay feedback test and an triangular test signal activation with same period of relay test. The series activation is used to tune each loop of the paired input and output. From the series activation, we can identify the nonlinear functions and the first order plus time delay models of the linear dynamic subsystem. The proposed identification method has several advantages that the frequency responses of the linear dynamic subsystem can be obtained and identified nonlinear functions can be used to eliminate the nonlinearity. In addition, when an actuator has its own saturation, we can exactly identify it. This characteristic can be applied to controller design for removing reset windup. The obtained full transfer function matrix can be used to reduce the time-consuming field experiments in the sequential loop closing method and to correct pairing. The identified transfer function matrix can also be used to design other model based control systems such as the decoupling control systems.

References

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