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PID Controller Tuning To Obtain Desired Closed Loop Responses for Cascade Control Systems

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A new method for PID controller tuning based on process models for cascaded control systems is proposed in this paper. The method consists of first finding the ideal controller that gives the desired closed loop response and then finding the PID approximation of the ideal controller by Maclaurin series. This method can be applied to any open loop stable processes. Furthermore, it enables us to tune the PID controllers both for the inner loop and the outer loop simultaneously while existing tuning methods tune the inner loop first and the outer loop next. Closed loop responses of cascade control loops tuned by the proposed method are compared with those of existing methods such as the frequency response method and the ITAE method. The results show that the proposed tuning method is superior to the existing methods.

1. Introduction

Cascade control is one of the most successful methods for enhancing single-loop control performance particularly when the disturbances are associated with the manipulated variable or when the final control element exhibits nonlinear behavior. This important benefit has led to the extensive use of cascade control in chemical process industries. It is well-known that control performance of the cascade control system largely depends on tuning of both inner and outer loops. However, information in the published literature on the tuning methods of cascade control appears to be rather limited. The frequency response methods (Jury, 1973; Hougen, 1979; Edgar et al., 1982) are usually employed to design the controllers because the open loop transfer function of the outer loop has higher order dynamics and/or time delay. However, a major impediment to the use of the frequency response methods in controller design has been the trial and error graphical calculations, which can be very tedious. Krishnaswamy (1990) provided tuning charts that predict the primary controller settings for minimizing ITAE criterion due to load disturbances on the secondary loop in cascade control systems. But the method is of limited use for the PI/P configuration and the first-order plus dead time (FOPDT) model over a limited range of model parameters. In addition, all of the foregoing approaches involve two steps for tuning cascade control systems: first the secondary controller is tuned on the basis of the dynamic model of the inner process; then, the primary controller is tuned on the basis of the dynamic model of the outer process including the secondary loop. Therefore, if the secondary controller is retuned for some reason, an additional identification step is essential for retuning the primary controller, which is often cumbersome in practice. In this paper, an efficient method of PID controller tuning for cascade control systems is proposed. The proposed method can be applied to any open loop processes.

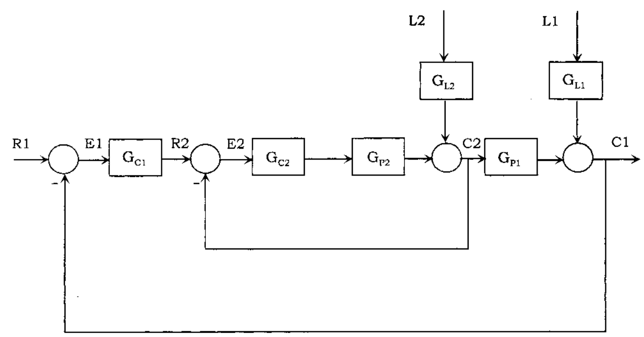

Figure 1. Block diagram of a cascade control system.

The contents of the paper are arranged as follows: A new theoretical development of tuning rules for general process models of cascade control systems by Maclaurin series is given first, followed by an example to illustrate the tuning procedure by the proposed method. Next the guidelines for closed loop time constants based on extensive simulation results are presented for performance and robustness of the system. Then, several simulation examples are provided to demonstrate the method and to compare its performance with those of other tuning methods. Finally, the last section deals with the conclusions.

2. Theory

Development of Tuning Rules for General Process Models. In cascade control systems, as shown in Figure 1, closed loop transfer functions for inner and outer loops are

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$$\frac{C_2}{R_2} = \frac{G_{C2}G_{P2}}{1 + G_{C2}G_{P2}} \tag{1}$$

$$\frac{C_1}{R_1} = \frac{G_{C1}G_{P1}\dfrac{C_2}{R_2}}{1 + G_{C1}G_{P1}\dfrac{C_2}{R_2}} = \frac{G_{C1}G_{P1}\dfrac{G_{C2}G_{P2}}{1 + G_{C2}G_{P2}}}{1 + G_{C1}G_{P1}\dfrac{G_{C2}G_{P2}}{1 + G_{C2}G_{P2}}} \tag{2a}$$

$$= \frac{G_{C1}G_{C2}G_{P1}G_{P2}}{1 + G_{C2}G_{P2} + G_{C1}G_{C2}G_{P1}G_{P2}} \tag{2b}$$

Here, the controllers G_{C1} and G_{C2} have to be designed to satisfy set-point tracking (R_1) and disturbance (L_1, L_2) regulating requirements.

(a) Design of Secondary Controller. A secondary controller has to be designed to reject the disturbances into the inner loop (L_2) stably as well as quickly. For this, the secondary variable should follow its set point as quickly as possible, but with little overshoot and oscillations. This requirement can be satisfied when the secondary controller is designed such that set-point tracking (C_2/R_2) gives a stable overdamping response. To tune the secondary controller to give such a set-point tracking response, the method by Lee et al. (1996, 1997, 1998) is used:

Consider a stable process model of the inner loop.

$$G_{P2}(s) = p_{2m}(s)p_{2A}(s) \tag{3}$$

where $p_{2m}(s)$ contains the invertible portion of the model and $p_{2A}(s)$ contains all the noninvertible portion. The noninvertible portion is typically chosen to be the all pass form as

$$\prod_{i,j}\left(\frac{-\tau_i s + 1}{\tau_i s + 1}\right)\left(\frac{\tau_j^2 s^2 - 2\tau_j\zeta_j s + 1}{\tau_j^2 s^2 + 2\tau_j\zeta_j s + 1}\right)e^{-Ts}$$
$$\tau_i,\ \tau_j > 0;\quad 0 < \zeta_j < 1$$

The requirement that $p_{2A}(0) = 1$ is necessary for the controlled variable to track its set point because this adds integral action to the controller.

Here, our purpose is to design the controller, G_{C2}, to make the closed loop transfer function of the inner loop, C_2/R_2. follow a desired closed loop response given by eq 4.

$$\frac{C_2}{R_2} = \frac{p_{2A}(s)}{(\lambda_2 s + 1)^{r2}} \tag{4}$$

The term $1/(\lambda_2 s + 1)^{r2}$ is an IMC filter with an adjustable time constant of the inner loop, λ_2, and $r2$ is chosen to ensure that the IMC controller (Rivera et al., 1986; Morari et al., 1989) is proper. Then, the feedback controller, G_{C2}, that gives the desired loop response is given by

$$G_{C2}(s) = \frac{q_2}{(1 - G_{P2}q_2)} = \frac{P_{2m}^{-1}(s)}{(\lambda_2 s + 1)^{r2} - P_{2A}(s)} \tag{5}$$

where q_2 is the IMC controller represented by $P_{2m}^{-1}(s)/(\lambda_2 s + 1)^{r2}$.

Since $p_{2A}(0)$ is 1, the controller G_{C2} can be expressed with an integral term as

$$G_{C2} \equiv f(s)/s \tag{6}$$

In order to approximate the above ideal controller to a PID controller, expanding $G_{C2}(s)$ in a Maclaurin series in s gives

$$G_{C2}(s) = \frac{1}{s}\left(f(0) + f'(0)s + \frac{f''(0)}{2}s^2 + ...\right) \tag{7}$$

It should be noted that the resulting controller has the proportional term, the integral term, and the derivative term, in addition to an infinite number of high-order derivative terms. Since the controller given by eq 7 is equivalent to the controller given by eq 5, the desired closed loop response can be perfectly achieved if all terms in eq 7 are implemented. In practice, however, it is impossible to implement the controller given by eq 7 because of the infinite number of high-order derivative terms. In fact, in the actual control situation low and middle frequencies are much more important than high frequencies, and only the first three terms in eq 7 are often sufficient to achieve the desired closed loop performance. The controller given by eq 7 can be approximated to the PID controller by using only the first three terms ($1/s$, 1, s) in eq 7 and truncating all other high-order terms (s^2, s^3, ...). The first three terms of the above expansion can be interpreted as the ideal PID controller given by

$$G_{C2}(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s + ...\right) \tag{8}$$

where

$$K_C = f'(0) \tag{8a}$$

$$\tau_I = f'(0)/f(0) \tag{8b}$$

$$\tau_D = f''(0)/2f'(0) \tag{8c}$$

The above eq 8 can be used to obtain the controller gain, the integral and derivative time constants as analytical functions of the process model parameters, and the closed loop time constant, λ_2.

The integral and derivative time constants (τ_I, τ_D) from eq 8 can have negative values for some complicated process models independent of the selection of filter time constant. In this case, the simple PID controller cascaded with a first-order lag of the form $1/(\alpha s + 1)$ or a second-order lag of the form $1/(\beta_2 s^2 + \beta_1 s + 1)$ is recommended (Lee et al., 1996, 1997, 1998).

(b) Design of Primary Controller. The PID controller tuned by the above procedure gives the closed loop response sufficiently close to the desired response (Lee et al., 1996, 1997, 1998). Thus, we can assume the closed loop transfer function for the inner loop, C_2/R_2, can be approximately represented by eq 4 with sufficient precision. Then, the term C_2/R_2 in eq 2a is substituted with eq 4.

$$\frac{C_1}{R_1} = \frac{G_{C1}G_{P1}\dfrac{P_{2A}(s)}{(\lambda_2 s + 1)^{r2}}}{1 + G_{C1}G_{P1}\dfrac{P_{2A}(s)}{(\lambda_2 s + 1)^{r2}}} \tag{9}$$

Therefore, the process model of the outer loop is considered as

$$G_1(s) = G_{P1}\frac{P_{2A}(s)}{(\lambda_2 s + 1)^{r2}} \tag{10}$$

Now, consider a stable process model of the outer loop of the form

$$G_1(s) = p_{1m}(s)\, p_{1A}(s) \tag{11}$$

where $p_{1m}(s)$ contains the invertible portion of the model, whereas $p_{1A}(s)$ contains all the noninvertible portion with the all pass form. Here, our purpose is also to design the controller, G_{C1}, so that the closed loop transfer function of the outer loop, C_1/R_1, has the form given by

$$\frac{C_1}{R_1} = \frac{P_{1A}(s)}{(\lambda_1 s + 1)^{r1}} \tag{12}$$

Then, the controller transfer function of the outer loop is represented by

$$G_{C1} = \frac{q_1}{1 - G_1 q_1} = \frac{P_{1m}{}^{-1}(s)(\lambda_2 s + 1)^{r2}}{P_{2A}(s)((\lambda_1 s + 1)^{r1} - P_{1A}(s))} \tag{13}$$

where q_1 is the IMC controller of the outer loop represented by $P_{1m}{}^{-1}(s)/(\lambda_1 s + 1)^{r1}$. In the same manner described in the earlier section, the ideal controller G_{C1} can also be approximated to a PID controller form.

Example of Tuning Rule for FOPDT Model. Since the most commonly used approximate model for chemical processes is the first-order plus dead time (FOPDT) model, a process with an FOPDT model both for inner and outer loops is considered as an example. Suppose a process of which models for inner and outer loops are given by

$$G_{P2}(s) = \frac{K_2 e^{-\theta_2 s}}{\tau_2 s + 1}, \quad G_{P1}(s) = \frac{K_1 e^{-\theta_1 s}}{\tau_1 s + 1} \tag{14}$$

Then, the model of the inner loop can be decomposed as

$$G_2(s) = p_{2m}(s)\, p_{2A}(s) = \frac{K_2}{\tau_2 s + 1} e^{-\theta_2 s} \tag{15}$$

Specifying a desired closed loop response as

$$\frac{C_2}{R_2} = \frac{P_{2A}}{(\lambda_2 s + 1)^{r2}} = \frac{e^{-\theta_2 s}}{\lambda_2 s + 1}$$

the ideal controller of the inner loop is given as

$$G_{C2}(s) = \frac{P_{2m}{}^{-1}(s)}{(\lambda_2 s + 1)^{r2} - P_{2A}(s)} = \frac{\tau_2 s + 1}{K_2(\lambda_2 s + 1 - e^{-\theta_2 s})} \tag{16}$$

Evaluating the PID parameters from eq 7 gives

$$K_{C2} = \frac{\tau_2 + \dfrac{\theta_2{}^2}{2(\lambda_2 + \theta_2)}}{K_2(\lambda_2 + \theta_2)}, \quad \tau_{I2} = \tau_2 + \frac{\theta_2{}^2}{2(\lambda_2 + \theta_2)},$$
$$\tau_{D2} = \frac{\theta_2{}^2}{6(\lambda_2 + \theta_2)}\left[3 - \frac{\theta_2}{\tau_2 + \dfrac{\theta_2{}^2}{2(\lambda_2 + \theta_2)}}\right] \tag{17}$$

Further, the model of the outer loop is given by

$$G_1(s) = G_{P1}\frac{P_{2A}(s)}{(\lambda_2 s + 1)^{r2}} = \frac{K_1 e^{-\theta_1 s}}{\tau_1 s + 1}\frac{e^{-\theta_2 s}}{\lambda_2 s + 1} \tag{18}$$

Thus, $P_{1m}(s)$ and $P_{1A}(s)$ are given as

$$G_1(s) = p_{1m}(s)\, p_{1A}(s) = \frac{K_1}{(\tau_1 s + 1)(\lambda_2 s + 1)} e^{-(\theta_1 + \theta_2)s} \tag{19}$$

Similarly, specifying a desired closed loop response as

$$\frac{C_1}{R_1} = \frac{P_{1A}(s)}{(\lambda_1 s + 1)^r} = \frac{e^{-(\theta_1 + \theta_2)s}}{(\lambda_1 s + 1)}$$

the ideal controller of the outer loop is given as

$$G_{C1} = \frac{P_{1m}{}^{-1}(s)(\lambda_2 s + 1)^{r2}}{P_{2A}(s)((\lambda_1 s + 1)^{r1} - P_{1A}(s))} = \frac{(\tau_1 s + 1)(\lambda_2 s + 1)}{K_1 e^{-\theta_2 s}(\lambda_1 s + 1 - e^{-(\theta_1 + \theta_2)s})} \tag{20}$$

Evaluating the PID parameters from eq 7 gives

$$K_{C1} = \frac{\tau_{I1}}{K_1(\lambda_1 + \theta_1 + \theta_2)}$$
$$\tau_{I1} = \tau_1 + \lambda_2 + \frac{(\theta_1 + \theta_2)^2}{2(\lambda_1 + \theta_1 + \theta_2)}$$
$$\tau_{D1} = \frac{\lambda_2 \tau_1 - \dfrac{(\theta_1 + \theta_2)^3}{6(\lambda_1 + \theta_1 + \theta_2)}}{\tau_{I1}} + \frac{(\theta_1 + \theta_2)^2}{2(\lambda_1 + \theta_1 + \theta_2)} \tag{21}$$

The resulting tuning rule for FOPDT models is summarized in Table 1. The results can be directly used for any cascade control mode. For example, in order to tune a P only controller, we use only the K_c term in Table 1 just by excluding integral and derivative terms.

Although only the case of a FOPDT model is introduced as an example, it should be noted that this approach can be directly applied to any other complex process models. Tuning rules for SOPDT models are also listed in Table 1.

Guideline for Closed Loop Time Constants λ_1 and λ_2. In the proposed approach, λ_1 and λ_2 are used as main parameters for adjusting the speeds of closed loop response. λ_1 and λ_2 are chosen so as to provide good performance and robustness. The disturbance rejection of the inner loop mainly depends on the secondary controller while the disturbance rejection and set-point tracking of the outer loop depend on the primary

Table 1. Resulting Tuning Rules for FOPDT and SOPDT Models

process	process model	reference trajectory	K_C	τ_I	τ_D
FOPDT (inner loop)	$G_{P2} = \dfrac{K_2 e^{-\theta_2 s}}{\tau_2 s + 1}$	$\dfrac{C_2}{R_2} = \dfrac{e^{-\theta_2 s}}{\lambda_2 s + 1}$	$\dfrac{\tau_I}{K_2(\lambda_2 + \theta_2)}$	$\tau_2 + \dfrac{\theta_2^{\,2}}{2(\lambda_2 + \theta_2)}$	$\dfrac{\theta_2^{\,2}}{6(\lambda_2 + \theta_2)}\left[3 - \dfrac{\theta_2}{\tau_I}\right]$
SOPDT (inner loop)	$G_{P2} = \dfrac{K_2 e^{-\theta_2 s}}{(\tau^2 s^2 + 2\xi\tau s + 1)}$	$\dfrac{C_2}{R_2} = \dfrac{e^{-\theta_2 s}}{\lambda_2 s + 1}$	$\dfrac{\tau_I}{K_2(\lambda_2 + \theta_2)}$	$2\xi\tau + \dfrac{\theta_2^{\,2}}{2(\lambda_2 + \theta_2)}$	$\dfrac{\tau^2 - \dfrac{\theta_2^{\,3}}{6(\lambda_2 + \theta_2)}}{\tau_I} + \dfrac{\theta_2^{\,2}}{2(\lambda_2 + \theta_2)}$
FOPDT (outer loop)	$G_{P1} = \dfrac{K_1 e^{-\theta_1 s}}{\tau_1 s + 1}$	$\dfrac{C_1}{R_1} = \dfrac{e^{-(\theta_1+\theta_2)s}}{\lambda_1 s + 1}$	$\dfrac{\tau_I}{K_1(\lambda_1 + \theta_1 + \theta_2)}$	$\tau_1 + \lambda_2 + \dfrac{(\theta_1 + \theta_2)^2}{2(\lambda_1 + \theta_1 + \theta_2)}$	$\dfrac{\lambda_2\tau_1 - \dfrac{(\theta_1 + \theta_2)^3}{6(\lambda_1 + \theta_1 + \theta_2)}}{\tau_I} + \dfrac{(\theta_1 + \theta_2)^2}{2(\lambda_1 + \theta_1 + \theta_2)}$
SOPDT (outer loop)	$G_{P1} = \dfrac{K_1 e^{-\theta_1 s}}{\tau^2 s^2 + 2\xi\tau s + 1}$	$\dfrac{C_1}{R_1} = \dfrac{e^{-(\theta_1+\theta_2)s}}{\lambda_1 s + 1}$	$\dfrac{\tau_I}{K_1(\lambda_1 + \theta_1 + \theta_2)}$	$2\tau\xi + \lambda_2 + \dfrac{(\theta_1 + \theta_2)^2}{2(\lambda_1 + \theta_1 + \theta_2)}$	$\dfrac{\tau^2 + 2\tau\xi\lambda_2 - \dfrac{(\theta_1 + \theta_2)^3}{6(\lambda_1 + \theta_1 + \theta_2)}}{\tau_I} + \dfrac{(\theta_1 + \theta_2)^2}{2(\lambda_1 + \theta_1 + \theta_2)}$

Table 2. Tuning Values by the Proposed Method and the Frequency Response Method for Example 1

	inner loop controller	outer loop controller
frequency response method (PI/P mode)	$K_{C2} = 4$	$K_{C1} = 3.5$, $\tau_{I1} = 5.3$
proposed method (PI/P mode)	$K_{C2} = 5$	$K_{C1} = 6.2$, $\tau_{I1} = 6.2$
proposed method (PID/PID mode)	$K_{C2} = 5$, $\tau_{I2} = 1$, $\tau_{D2} = 0$	$K_{C1} = 6.2$, $\tau_{I1} = 6.2$, $\tau_{D1} = 1.484$

controller. As a result, λ_1 and λ_2 can be chosen independently in most cases. The guidelines for λ_1 and λ_2 are studied especially for the case of the FOPDT model. In practice, the closed loop band width is usually chosen such that it does not exceed 10 times more than the open loop band width (Morari et al., 1989). Therefore, as a rough guideline, it is recommended that at least λ_1 and λ_2 are 10 times less than those of corresponding open loop time constants. The optimal value of λ is a function of process dead time θ. Specifying one value of λ/θ for any FOPDT model results in an identical response when time is scaled by θ, regardless of K, θ, and τ (Morari et al., 1989). Extensive simulation was done to find the best ratios $\lambda_1/(\theta_1 + \theta_2)$ and λ_2/θ_2 in the sense of robustness and performance. As a result, $\lambda_1/(\theta_1 + \theta_2) = 0.5$ and $\lambda_2/\theta_2 = 0.5$ are recommended.

3. Simulation Study

To evaluate the proposed tuning rule, simulations for typical cases were done. All the simulations were performed using MATLAB (control system design and simulation software, 1993).

Example 1. Firstly, the following process model (Seborg et al., 1989) was studied.

$$G_{P1} = \frac{4}{(2s+1)(4s+1)},\quad G_{P2} = \frac{5}{s+1},\quad G_{L1} = \frac{1}{3s+1},\quad G_{L2} = 1,\quad G_{m1} = 0.05,\quad G_{m2} = 0.2 \quad (22)$$

The PID controllers for inner and outer loops for the above process were tuned by the proposed tuning rules. The closed loop time constants for inner and outer loops were chosen as $\lambda_1 = 1$ and $\lambda_2 = 0.2$. Two control modes (PID/PID mode and PI/P mode) were tested for the proposed method. The results were compared with those by the frequency method (Seborg et al., 1989). The resulting PID parameters are listed in Table 2. Since the PID controllers in cascade control should be tuned considering all the closed loop performances both for set-point tracking (C_1/R_1) and disturbance rejection (C_1/L_1 and C_1/L_2), the tuning methods were tested in terms of all these performances. Figures 2, 3, and 4 show the closed loop responses tuned by the proposed method and the frequency response method for the unit step change in L_2, L_1, and R_1, respectively. The results shown in the figures illustrate the superior performance of the proposed method.

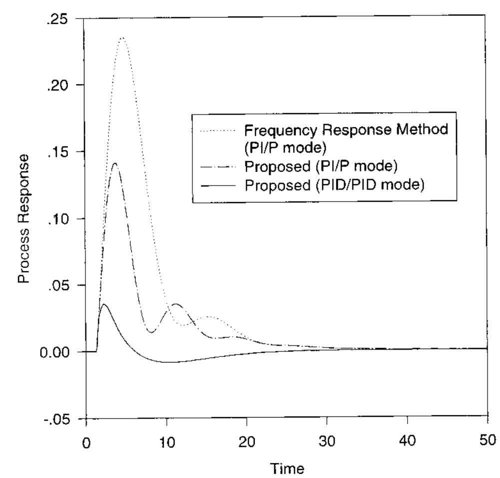

Figure 2. Closed loop response due to load change (C_1/L_2) for example 1.

Example 2. Since many chemical processes can be represented by FOPDT models, the following process

Table 3. Tuning Values by the Proposed Method and the ITAE Method for Example 2

	inner loop controller	outer loop controller
ITAE method (PI/P mode)	$K_{C2} = 2.978$	$K_{C1} = 7.3$, $\tau_{I1} = 200$
proposed method (PI/P mode)	$K_{C2} = 3.444$	$K_{C1} = 5.83$, $\tau_{I1} = 105$
proposed method (PID/PID mode)	$K_{C2} = 3.444$, $\tau_{I2} = 20.666$, $\tau_{D2} = 0.6451$	$K_{C1} = 5.83$, $\tau_{I1} = 105$, $\tau_{D1} = 4.8$

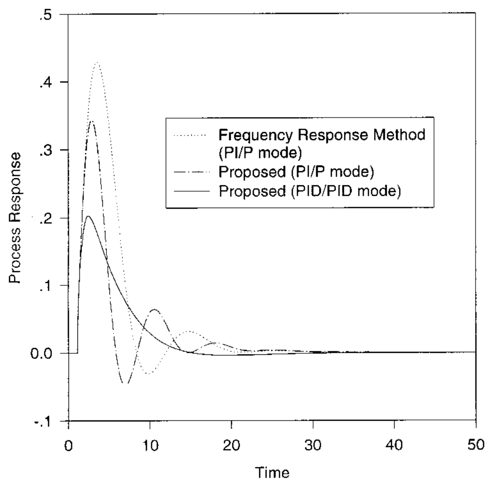

Figure 3. Closed loop response due to load change (C_1/L_1) for example 1.

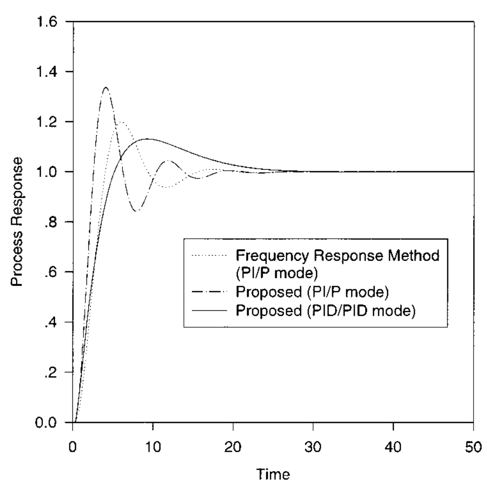

Figure 4. Closed loop response due to set point change (C_1/R_1) for example 1.

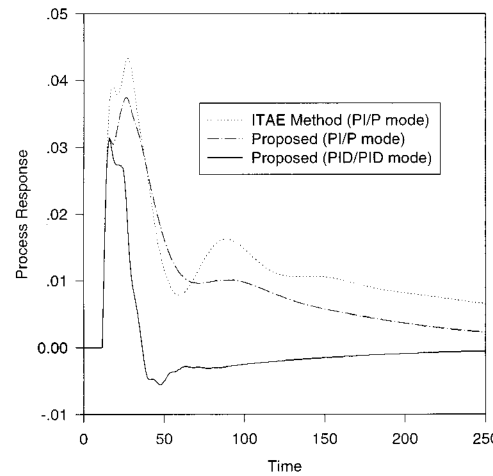

Figure 5. Closed loop response due to load change (C_1/L_2) for example 2.

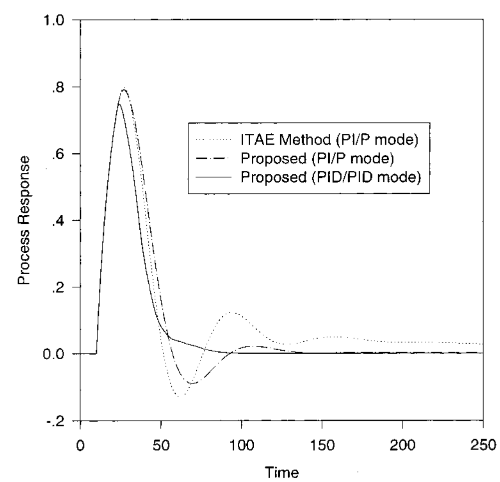

Figure 6. Closed loop response due to load change (C_1/L_1) for example 2.

(Krishnaswamy, 1990) was studied as the second example.

$$G_{P1} = \frac{e^{-10s}}{100s + 1}, \quad G_{P2} = \frac{2e^{-2s}}{20s + 1}, \quad G_{L1} = \frac{e^{-10s}}{10s + 1}, \quad ,G_{L2} = 1 \quad (23)$$

The PID controllers were tuned by the proposed method with $\lambda_1 = 6$ and $\lambda_2 = 1$. The values of simulation results were compared with those by the ITAE method (Krishnaswamy, 1990). In Table 3, the PID tuning values used in the simulation are presented. Figures 5, 6, and 7 show the closed loop responses tuned by the proposed method and the ITAE method for the unit step changes in L_2, L_1, and R_1, respectively. The results shown in the figures also illustrate the superior performance of the proposed method. Another interesting point to note is the inferior performance by the ITAE method even in the case of load disturbance on the secondary loop. This may be due to two possible reasons: the simple PI/P control mode used in the ITAE method (note that the PID/PID control mode was used in simulation by the proposed method) and local optima in ITAE optimization, which is often the case in nonlinear optimization problems. In many cases including flow loops, the derivative mode is excluded in the secondary controller. To evaluate the effect of the type of control mode, every combination of control modes was considered. The responses of various cascade control modes are presented in Figure 8 for the proposed method and the ITAE method. The figure reveals that the superior performance by the proposed method compared to the ITAE method is not only due to the effect of the control mode but also due to the local optimum problem in ITAE optimization. As shown in the figure, the response by the proposed tuning method shows still better performance than that by the ITAE method. It is also shown that the improvement of control performance is mainly achieved by the properly tuned integral term in the secondary controller and the derivative term in the primary controller.

Table 4. Tuning Values by the Proposed Method and the ITAE Method for Example 3

	inner loop controller	outer loop controller
ITAE method (PI/P mode)	$K_{C2} = 0.6625$	$K_{C1} = 0.1373$, $\tau_{I1} = 485.37$
proposed method (PI/P mode)	$K_{C2} = 0.883$	$K_{C1} = 0.09$, $\tau_{I1} = 90.53$
proposed method (PID/PID mode)	$K_{C2} = 0.883$, $\tau_{I2} = 14.5$, $\tau_{D2} = 1.117$	$K_{C1} = 0.09$, $\tau_{I1} = 90.53$, $\tau_{D1} = 18.2$

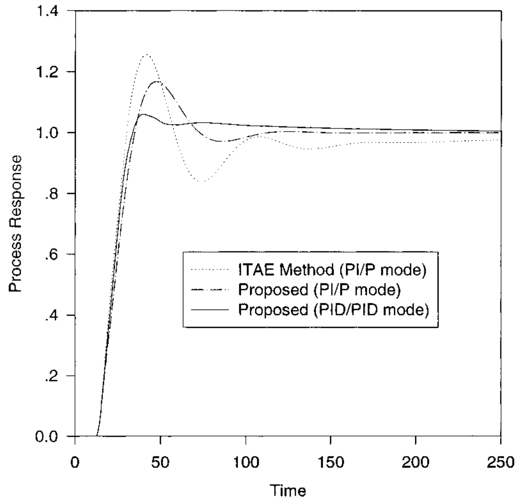

Figure 7. Closed loop response due to set point charge (C_1/R_1) for example 2.

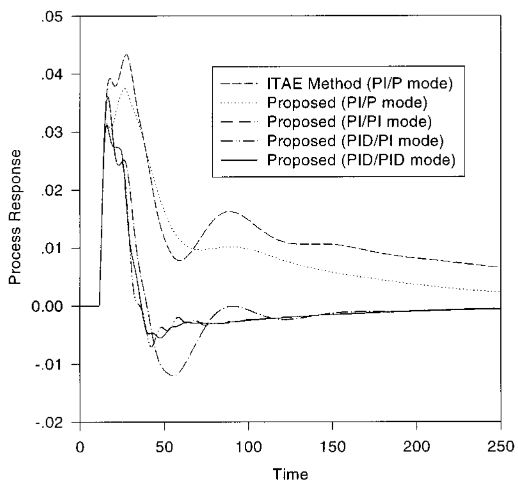

Figure 8. Comparison of the closed loop responses due to load change (C_1/L_2) in the PI/P, PI/PI, PID/PI, and PID/PID control modes.

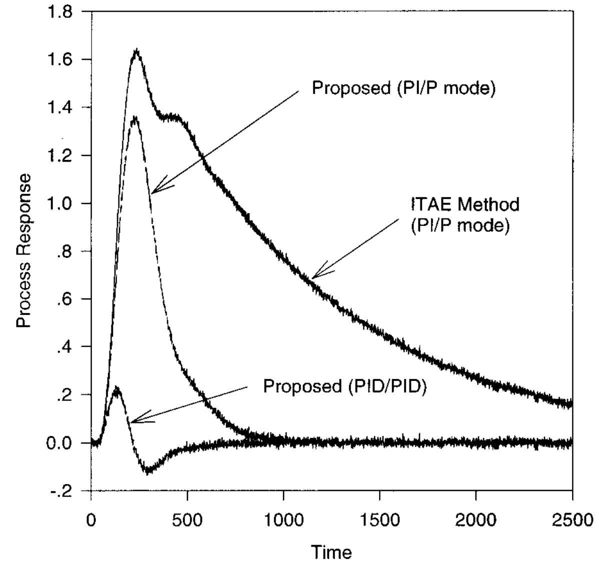

Figure 9. Closed loop response due to load change (C_1/L_2) for example 3.

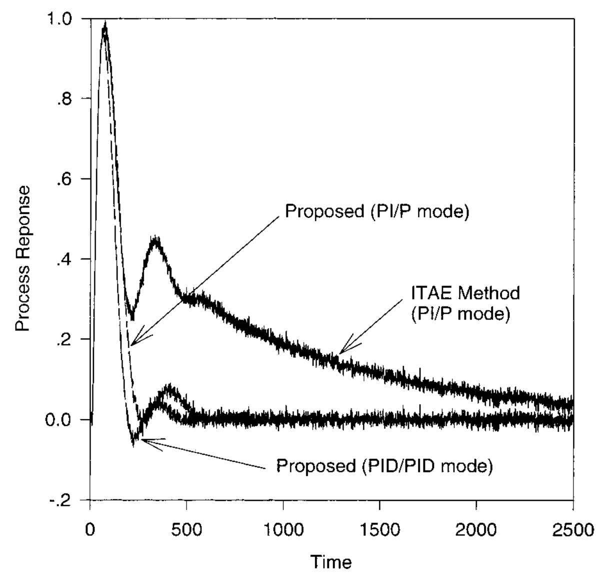

Figure 10. Closed loop response due to load change (C_1/L_1) for example 3.

Example 3. To evaluate the robustness against structural mismatch in the plant and the model, the following complicated process was tested.

$$G_{P1} = \frac{10(-5s + 1)e^{-5s}}{(30s + 1)^3(10s + 1)^2}, \quad G_{P2} = \frac{3e^{-3s}}{13.3s + 1},$$
$$G_{L1} = \frac{e^{-10s}}{100s^2 + 20s + 1}, \quad ,G_{L2} = \frac{1}{100s + 1} \quad (24)$$

We added white noises to C_2 and C_1 to represent real process measurements. The variances of the noises are 1E-4 and 1E-4 in measurements, respectively. We identified the processes in the inner loop and the outer loop with the FOPDT model. The models were obtained by minimization of the squared error between process output data and model output data. We obtained the process models as follows:

$$G_{m1} = \frac{10.2e^{-61.71s}}{66.49s + 1}, \quad G_{m2} = \frac{2.988e^{-3.66s}}{13.28s + 1} \quad (25)$$

The PID controllers were tuned by the proposed method with $\lambda_1 = 30.85$ and $\lambda_2 = 1.83$. Note that the closed loop time constants λ_1 and λ_2, were chosen by the guidelines $\lambda_1/(\theta_1 + \theta_2) = 0.5$ and $\lambda_2/\theta_2 = 0.5$. The values of simulation results were also compared with those by the ITAE method (Krishnaswamy, 1990). The PID tuning values used in the simulation are presented in Table 4. Figures 9, 10, and 11 show the closed loop responses tuned by the proposed method and the ITAE method for the unit step changes in L_2, L_1, and R_1, respectively.

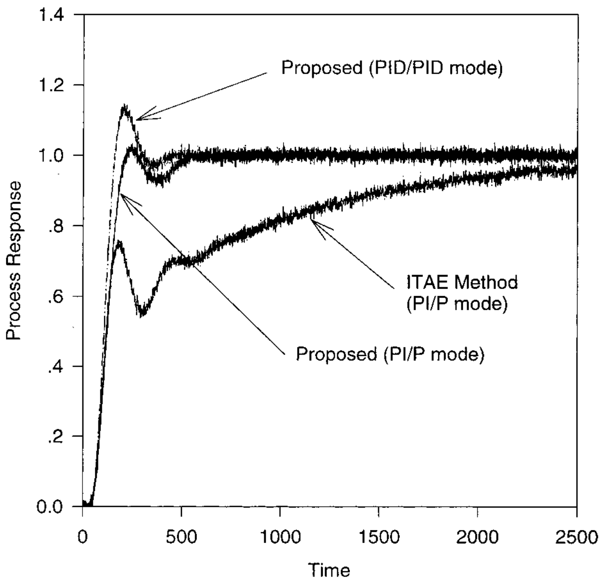

Figure 11. Closed loop response due to set point change (C_1/R_1) for example 3.

The superior performance of the proposed method is readily apparent.

4. Conclusions

A new method for PID controller tuning for cascade control systems was proposed. The tuning rule is based on the process model and the desired closed loop response. The ideal controller which can give the desired closed loop response is found and the PID approximation of the ideal controller is obtained by taking the first three terms from Maclaurin series expansion of the ideal controller. Extensive simulation study illustrates that the proposed method gives better performance compared with the existing methods. In addition to this main benefit, the method has several advantages: it is simple and easy to use because the tuning parameters are in analytical form; the tuning of inner and outer loop controllers can be done simultaneously, and no additional identification step is required even when the secondary controller is retuned because the proposed tuning method is based on the model parameters of the process; the cascade control system can be tuned to meet the specifications of both the inner and outer loops because the proposed method has two adjustable parameters.

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Notation

C = controlled variable
E = error
M = manipulated variable
K_C = gain, proportional tuning parameter
K = process gain
$P_m(s)$ = portion of a process model inverted by the controller; it must be minimum phase
$P_A(s)$ = portion of a process model **not** inverted by the controller; it is usually a nonminimum phase (i.e. contains dead times and/or right half plane zeros)
r = relative order of $P_m(s)$
R = set point

Greek Letters

ξ = damping ratio, dimensionless
τ_I = reset time, integral tuning parameter
τ_D = derivative tuning parameter
λ = time constant of reference trajectory
τ = process model time constant
θ = dead time

Acronyms

IMC = internal model control
ISE = integral of the square error
ITAE = integral of the time-weighted absolute error
PID = proportional-integral-derivative

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