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Robust PID tuning for Smith predictor in the presence of model uncertainty

Dongkwon Lee [a], Moonyong Lee [a,*], Suwhan Sung [b], Inbeum Lee [b]

[a] School of Chemical Engineering and Technology, Yeungnam University, Kyongsan 712-749, Korea
[b] Department of Chemical Engineering, Pohang University of Science and Technology, Pohang 790-784, Korea

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Abstract

This paper presents robust PID tuning for the Smith predictor in the presence of model uncertainty. The concept of the equivalent gain plus time delay (EGPTD) is introduced to incorporate robust stability in PID tuning of the Smith predictor. In particular, an application is developed for the robust tuning of the first order plus time delay (FOPTD) system and the second order plus time delay (SOPTD) system because the systems have been used extensively to describe chemical processes. The proposed tuning method can cope with simultaneous uncertainties in all parameters of the model in an efficient manner. Another important and attractive feature of the method is that it can utilize many currently available PID tuning rules. Simulation results are provided to demonstrate the availability of the method.

Keywords: Robust PID tuning; Smith predictor; Model uncertainty

Time delays occur frequently in process control problems due to the distance velocity lags, recycle loops, and composition analysis loops. The presence of the time delay in a process greatly complicates the analytical aspects of controller design and makes satisfactory control more difficult to achieve. Smith predictor [1] or dead time compensator has attracted many researchers and control engineers as the one of the most effective control strategies to incorporate the time delay. However, although the Smith predictor offers potential improvement in the closed loop performance of the processes with large time delays, its application in the industry has been limited due to several problems. One of the main problems is its sensitivity to modeling errors. Like other model-based control systems, the Smith predictor requires a good model of the process. In the face of inevitable mismatches between the model and the actual process, the closed loop performance can be very poor. Even if the Smith predictor is nominally stable, it can be destabilized even by infinitesimal perturbation in the process dynamics [2].

For this reason, recently, research effort has been focused on the robust tuning issues of the Smith predictor. Ioannides et al. [3] plotted stability boundaries as functions of error in a single plant parameter. Brosilow [4] proposed a tuning method for the uncertainty in one model parameter. Owens and Raya [5] addressed the effect of additive plant/model mismatch on the robust stability of the Smith predictor under the uncertainty in the time delay. Morari and Zafiriou [6] presented the tuning guideline based on one degree of freedom controller which was tuned for tracking set-point by considering plant/model mismatches in the time delay. Santacesaria and Scattolini [7] proposed a method for selection of the closed loop band-width under the uncertainty bound on the time delay. Palmor and Blau [8] developed a robust tuning rule considering the modeling error in the time delay. However, all these researches do not address simultaneous uncertainties in process gain, time constant, and time delay. Laughlin et al. [9] presented an iterative tuning method for robust performance by mapping the parametric uncertainty to convex hulls on the complex plane. Lee et al. [10] also proposed a two-step approach for robust tuning incorporating simultaneous uncertainties. But the procedures are computationally intensive including a trial–error step and cumbersome to implement in a practical way.

In this article, we propose a robust PID tuning method for the Smith predictor system under the model

* Corresponding author. Tel.: 82-53-810-2512; fax: 82-53-814-8790; e-mail: mynlee@ynucc.yeungnam.ac.kr

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uncertainty. In the proposed method, the effect of simultaneous uncertainties in all the parameters of the model can be incorporated in an efficient manner by introducing the concept of the equivalent gain plus time delay (EGPTD). In particular, a robust tuning for the processes with the first order plus time delay (FOPTD) model and the second order plus time delay (SOPTD) model is considered because the two types of models are widely accepted to capture the essential dynamics of the majority of loops in chemical processes. It is also important that the approximation based on the EGPTD does not change the structure of the total transfer function while simultaneous uncertainties in all parameters are taken into consideration. As a consequence, the proposed method can utilize many currently available PID tuning rules for the FOPTD system and the SOPTD system such as the Z–N method (Ziegler and Nichols [11]), the IMC-PID method (Rivera et al. [12]), the ITAE method (Lopez et al. [13]), and the DCLR method (Lee et al. [14]).

1. Smith predictor

Consider a SISO, open loop stable linear process with transfer function $G_p(s)$. The Smith predictor system is shown in Fig. 1. $G_m^*(s)$ represents a delay-free part of a model and $G_m(s)$ denotes the model. $G_c(s)$ is a conventional controller and usually taken to be a PID controller. The closed loop transfer function is then given by

$$\frac{y(s)}{y_{sp}(s)}=\frac{G_c(s)G_p(s)}{1+G_c(s)(G_m^*(s)+G_p(s)-G_m(s))} \tag{1}$$

When the model is perfect $[G_p(s)=G_m(s)]$, the time delay would then be eliminated from the characteristic equation of the closed system, and thus the controller $G_c(s)$ can be designed as if the system is delay-free. The achievable performance can thus be greatly improved over a conventional system by allowing high controller gain. However, in the face of even small modeling error, which is inevitable in the real world, the Smith predictor may give very poor performance and lose stability because the modeling error term $G_p(s)$–$G_m(s)$ is significantly amplified by the high controller gain. For this reason, in addition to nominal stability and performance, the controller $G_c(s)$ should be tuned in such a way that robust stability and performance of the system can be ensured.

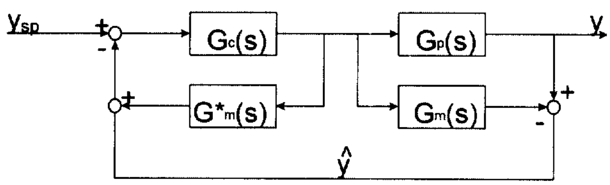

Fig. 1. Block diagram of the Smith predictor system.

2. Concept of equivalent gain plus time delay (EGPTD)

In this section, we introduce the concept of the EGPTD in order to deal with the effect of simultaneous uncertainties in the model parameters. The key idea is to force the complicate term $G_m^*(s)+G_p(s)-G_m(s)$ in the characteristic equation to the simple form consisting of a delay free part and an equivalent gain plus time delay (EGPTD) through adequate approximation as follows:

$$G_m^*(s)+G_p(s)-G_m(s)\approx G_m^*(s)K_{eq}e^{-\theta_{eq}s}=G_w(s) \tag{2}$$

where K_{eq} and θ_{eq} denote the equivalent gain and the equivalent time delay, respectively. There are several reasons to adopt the EGPTD for approximation: first, it inherently provides fairly good coincidence with the corresponding transfer function over low and middle frequency ranges which are important in process control situation; second, in high frequency range it gives at least a conservative approximation so that the robustness is ensured; third, the structure of the model approximated by the EGPTD remains same as that of the process model so that existing tuning rules for the process model can be applied for robust Smith system design. The last property offers an important advantage especially in the case where the process model is the FOPTD model and the SOPTD model because many efficient PID tuning rules [12–15] for the FOPTD model and the SOPTD model are currently available and thus can be directly used for tuning of the controller $G_c(s)$ in the Smith system.

The approximating transfer function $G_w(s)$ is taken to be the worst case process so that robust stability is ensured for all members of a family of possible processes. It is obvious that the controller stabilizing the worst case would stabilize all possible processes. Using $G_w(s)$, the closed loop transfer function can be rewritten, as shown in Fig. 2, in terms of the usual feedback system as

$$\frac{y(s)}{y_{sp}(s)}=\frac{G_c(s)G_w(s)}{1+G_c(s)G_w(s)}G_w^{-1}(s)G_p(s) \tag{3}$$

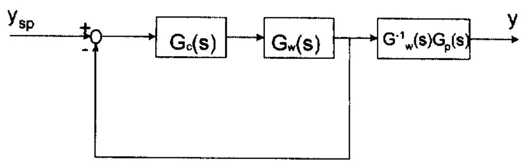

Fig. 2. Simplified block diagram of the Smith predictor system using EGPTD.

The net result of the introduction of $G_w(s)$ is therefore to convert the complicate model mismatch term to relatively simple form and thus to allow the controller to be easily tuned based only on the simple model. $G_p(s)$ is moved outside of the loop, where it has no effect on closed loop stability. Further, since the equivalent time delay is smaller than the actual time delay unless the uncertainty is extremely large, no causality problem occurs in the block $G_w^{-1}(s)G_p(s)$

2.1. Approximation by EGPTD

The EGPTD $K_{eq}e^{-\theta_{eq}s}$ corresponding to a general transfer function $G(s)$ can be easily obtained from Maclaurin series expansion.

First, expanding $G(s)$ in a Maclaurin series in s gives

$$G(s) = G(0) + \left.\frac{dG(s)}{ds}\right|_{s=0} s + \frac{1}{2} + \left.\frac{d^2G(s)}{ds^2}\right|_{s=0} s^2 + \ldots \quad (4)$$

Expanding the EGPTD candidate $K_{eq}e^{-\theta_{eq}s}$ in a Maclaurin series in s also gives

$$K_{eq}e^{-\theta_{eq}s} = K_{eq} - K_{eq}\theta_{eq}s + \frac{1}{2}K_{eq}\theta_{eq}^2 s^2 + \ldots \quad (5)$$

K_{eq} and θ_{eq} should be determined to approximate $G(s)$ as close as possible over important frequency range. Considering that in the process control situation the lower momentum terms are often more important than the higher ones, K_{eq} and θ_{eq} would be obtained by comparing the first term and the second term of (4) and (5), respectively. Thus,

$$K_{eq} = G(0) \quad (6)$$

$$\theta_{eq} = -\frac{1}{K_{eq}}\frac{dG(s)}{ds}|_{s=0} \quad (7)$$

As another way, comparison of the third terms of (4) and (5) gives the following result for the equivalent time delay

$$\theta_{eq} = \sqrt{\frac{1}{K_{eq}}\frac{d^2G(s)}{ds^2}|_{s=0}} \quad (8)$$

The way for the equivalent time delay calculation is chosen by considering a trade-off between performance and robustness: the EGPTD based on the lower momentum term yields better performance but less robustness in high frequency range. Besides, in particular case of the SOPTD model, the EGPTD model using (6) and (7) does not incorporate the modeling error in the time constant while that using (6) and (8) does.

2.2. Application of EGPTD to FOPTD case

To ensure robust stability, the EGPTD is chosen based on the worst case process which is certainly the case corresponding to the largest equivalent gain and time delay.

Dividing both sides of (2) by $G_m^*(s)$ yields

$$1 + \frac{G_p^*(s)}{G_m^*(s)}e^{-\theta_p s} - e^{-\theta_m s} \approx K_{eq}e^{-\theta_{eq}s} \quad (9)$$

Consider the model and the process are the FOPTD systems as

$$G_m(s) = \frac{K_m e^{-\theta_m s}}{\tau_m s + 1} \quad (10)$$

$$G_p(s) = \frac{K_p e^{-\theta_p s}}{\tau_p s + 1} \quad (11)$$

Then, we can obtain

$$1 + \frac{K_p}{K_m}\frac{\tau_m s + 1}{\tau_p s + 1}e^{-\theta_p s} - e^{-\theta_m s} \approx K_{eq}e^{-\theta_{eq}s} \quad (12)$$

Thus, the equivalent gain and time delay using (6) and (7) are

$$K_{eq} = \frac{K_p}{K_m} \quad (13)$$

$$\theta_{eq} = (\tau_p - \tau_m) + \theta_p - \frac{K_m}{K_p}\theta_m \quad (14)$$

It is often convenient to model processes with transfer functions containing real parameter uncertainties as in

$$\prod = \left\{ G(s) | G(s) = K\left[\frac{a_m s^m + a_{m-1}s^{m-1} + \ldots + a_1 s^1 + 1}{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s^1 + 1}\right]e^{-\theta s} \right\}$$
$$a_i \in [a_{imin}, a_{imax}], b_i \in [b_{imin}, b_{imax}], K \in [K_{\min}, K_{\max}], \theta \in [\theta_{\min}, \theta_{\max}] \quad (15)$$

Thus, in the case of the FOPTD, the parameter bounds are described by

$$K_m - \delta_K \le K_p \le K_m + \delta_K \quad (16)$$

$$\tau_m - \delta_\tau \le \tau_p \le \tau_m + \delta_\tau \quad (17)$$

$$\theta_m - \delta_\theta \le \theta_p \le \theta_m + \delta_\theta \quad (18)$$

Since the worst case should have the largest equivalent gain and time delay, the EGPTD of the worst case should be chosen as

$$K_{eq} = \frac{K_m + \delta_K}{K_m} \quad (19)$$

$$\theta_{eq} = \frac{\delta_K}{K_m + \delta_K}\theta_m + \delta_\theta + \delta_\tau \quad (20)$$

Once the EGPTD of the worst case is obtained, the controller $G_c(s)$ is tuned based only on corresponding $G_m^*(s)K_{eq}e^{-\theta_{eq}s}$. Further, as mentioned earlier, since $G_m^*(s)K_{eq}e^{-\theta_{eq}s}$ maintains the FOPTD form, any conventional PID tuning rules for the FOPTD model can be used to tune the controller.

2.3. Application of EGPTD to SOPTD case

The SOPTD case is also considered because of its relevance in process control. The same procedure used for the FOPTD case is applied to determine the EGPTD of the worst case process. Assume that the process and the model are given by

$$G_p(s) = \frac{K_p e^{-\theta_p s}}{a_p s^2 + b_p s + 1} \quad (21)$$

$$G_m(s) = \frac{K_m e^{-\theta_m s}}{a_m s^2 + b_m s + 1} \quad (22)$$

Furthermore, assume the parameters in the model are uncertain,

$$K_m - \delta_K \le K_p \le K_m + \delta_K \quad (23)$$

$$\theta_m - \delta_\theta \le \theta_p \le \theta_m + \delta_\theta \quad (24)$$

$$a_m - \delta_a \le a_p \le a_m + \delta_a \quad (25)$$

$$b_m - \delta_b \le b_p \le b_m + \delta_b \quad (26)$$

Substitution of (21) and (22) into (9) and expansion in a Maclaurin series gives

$$1+\frac{G_p^*(s)e^{-\theta_p s}}{G_m^*(s)} - e^{-\theta_m s} = \frac{K_p}{K_m}+\frac{K_p}{K_m}\left(b_m - b_p - \theta_p + \frac{K_m}{K_p}\theta_m\right) s + \frac{K_p}{2K_m}\left\{a_m - a_p + b_p(b_p - b_m) + 2(b_p - b_m)\theta_p + \theta_p^2 - \frac{K_m}{K_p}\theta_m^2\right\}s^2 + \ldots \quad (27)$$

From (6), (23), and (27), the equivalent gain corresponding to the worst case is

$$K_{eq} = \frac{K_m + \delta_K}{K_m} \quad (28)$$

Further, if we use (7), the equivalent time delay becomes

$$\theta_{eq} = \theta_p + b_p - b_m - \frac{K_m}{K_p}\theta_m \quad (29)$$

Thus, considering the uncertainty bounds given by (23)–(26), the equivalent time delay corresponding to the worst case is

$$\theta_{eq} = \delta_\theta + \delta_b + \frac{\delta_K}{K_m + \delta_K}\theta_m \quad (30)$$

On the other hand, if we use (8) for estimating the equivalent time delay, then

$$\theta_{eq}=\sqrt{\theta_p^2+2(b_p-b_m)\theta_p+a_m-a_p+b_p(b_p-b_m)-\frac{K_m}{K_p}\theta_m^2} \quad (31)$$

Thus, the equivalent time delay of the worst case becomes

$$\theta_{eq}=\sqrt{\frac{\delta_K}{K_m+\delta_K}\theta_m^2+2\theta_m(\delta_\theta + \delta_b)+\delta_\theta^2+2\delta_b\delta_\theta - \delta_a+(b_m + \delta_b)\delta_b} \quad (32)$$

It is noted that (30) does not contain the uncertainty effect of the parameter a_p because it is based on the first derivative value of the corresponding transfer function at zero frequency. Generally, if we use the high momentum term, the resulting equivalent time delay can incorporate the uncertainty effect in the parameter of the corresponding high order s term. However, the use of the high momentum term tends to increase value of the equivalent time delay and thus results in more conservative approximation.

Once the EGPTD of the worst case is determined, the controller can be robustly tuned using conventional tuning rules for the SOPTD model straightforwardly.

3. Simulation studies

To evaluate the proposed method for the conventional Smith Predictor (Fig. 1), we show now the following examples.

Example 1. Consider the FOPTD process and the nominal model as

$$G_p(s) = \frac{1.1e^{-1.1s}}{1.1s + 1},\ G_m(s) = \frac{1.0e^{-1.0s}}{1.0s + 1}$$

From (19) and (20), the corresponding EGPTD is obtained as $K_{eq}e^{-\theta_{eq}s} = 1.1e^{-0.29s}$. Thus, the approximating transfer function becomes

$$G_w(s) = \frac{1.1e^{-0.29s}}{1.0s + 1}$$

In order to compare the approximation precision of the EGPTD, the Bode diagram of $G_m^*(s) + G_p(s) - G_m(s)$ and the corresponding $G_m^*(s)K_{eq}e^{-\theta_{eq}s}$ are plotted in Fig. 3. As can be seen in the figure, the approximation by the EGPTD shows fairly good coincidence in low frequency range. In such a meaning, the approximation by the EGPTD is considered to be suitable to the case of control system where main properties are determined by the low frequency characteristic, which is often the case of process control. The magnitude of the EGPTD still shows good coincidence even in high frequency range. On the other hand, the phase angle of the EGPTD becomes more conservative as the frequency becomes high. Although losing the approximation precision in high frequency, the conservative approximation of the phase angle in high frequency ensures the control system tuned based on the EGPTD to have robust stability.

Fig. 4 compares the closed loop responses of the Smith predictor system tuned by the Palmor's method [8] and by the proposed method. Since the Palmor's method considers only the modeling error in the time delay, it can cause aggressive control action. The controller action can be amplified by the modeling error in the process gain as shown in the figure. In the proposed method, the conventional ITAE-PI tuning rule [13] was applied. Since the proposed method considers simultaneous uncertainties in the process gain, the time constant, and the time delay systematically, it results in better control performance.

Example 2. Consider the SOPTD process and the nominal model as

$$G_p(s) = \frac{1.2e^{-1.2s}}{1.2s^2 + 2.4s + 1},\ G_m(s) = \frac{1.0e^{-1.0s}}{1.0s^2 + 2.0s + 1}$$

The corresponding EGPTD is obtained from (30) as $K_{eq}e^{-\theta_{eq}s} = 1.2e^{-0.767s}$. Thus,

$$G_w(s) = \frac{1.2e^{-0.767s}}{1.0s^2 + 2.0s + 1}$$

On the other hand, the EGPTD based on (32) is obtained as $K_{eq}e^{-\theta_{eq}s} = 1.2e^{-1.65s}$ and

$$G_w(s) = \frac{1.2e^{-1.65s}}{1.0s^2 + 2.0s + 1}$$

Fig. 5 shows the Bode plot of $G_m^*(s) + G_p(s) - G_m(s)$ and the corresponding $G_m^*(s)K_{eq}e^{-\theta_{eq}s}$ from (30) and (32). The same conclusion as the FOPTD case shown in Example 1 can be drawn from the figure. Further, as expected, the EGPTD based on the high momentum term gives more conservative approximation for the

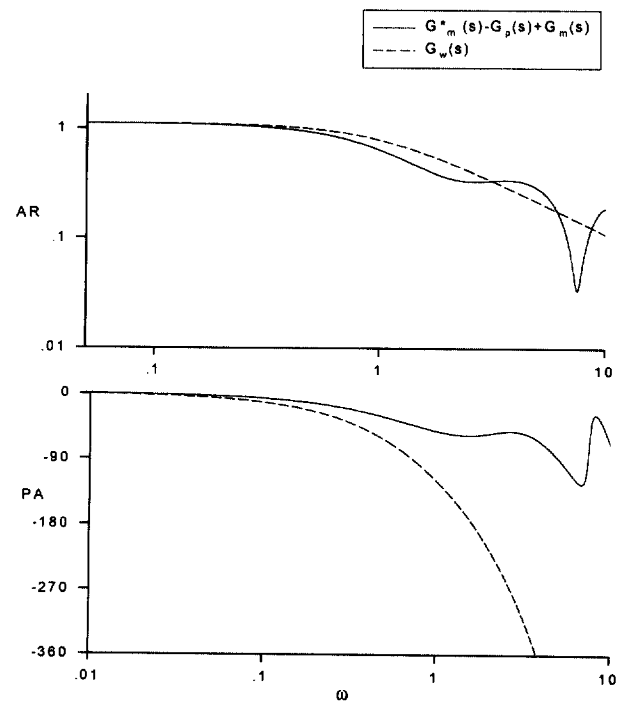

Fig. 3. Bode diagram of the transfer functions (Example 1).

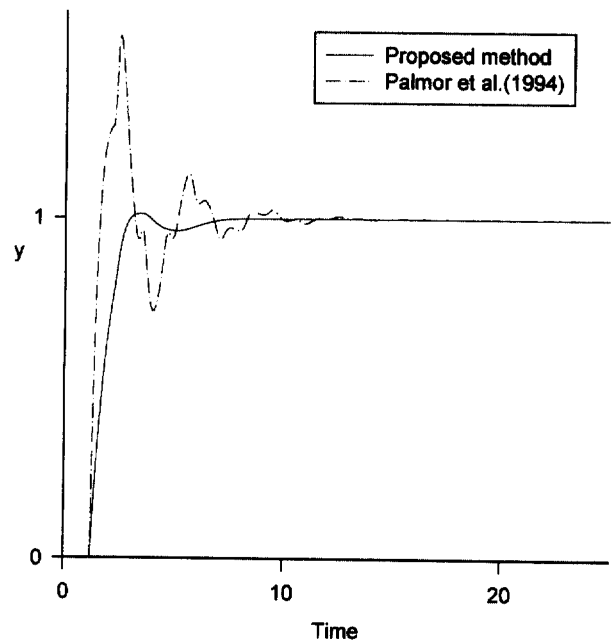

Fig. 4. Comparison of the closed loop responses for a unit step change in set-point (Example 1).

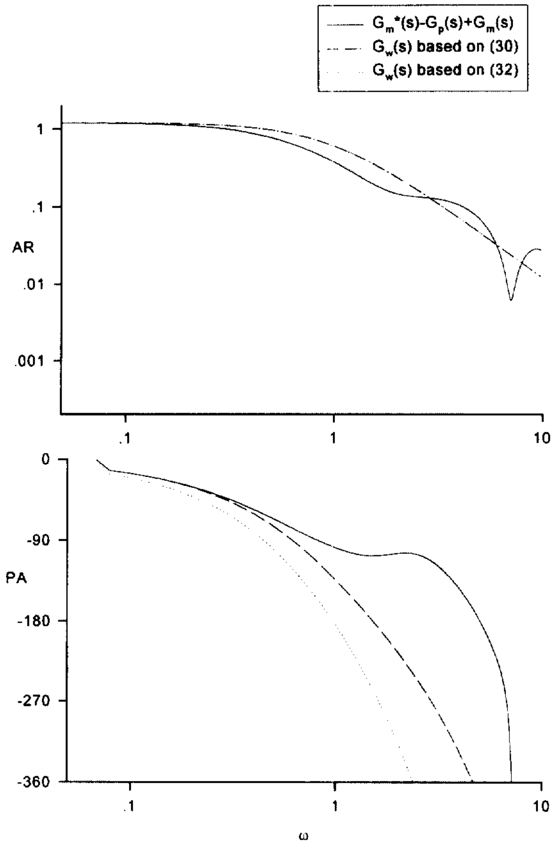

Fig. 5. Bode diagram of the transfer functions (Example 2).

phase angle than that based on the low momentum term. It implies that the controller based on (32) gives more sluggish but robust response than the one based on (30) as illustrated in Fig. 6. Empirically, the modeling error in the parameter for high order s term does not significantly affect the closed loop stability unless it is excessively large, and thus the use of (30) is recommend for robust performance.

Example 3. To evaluate the effect of structural uncertainty in process-model, a complicate high order process is considered. Assume a process and a simplified model are

$$G_{p-actual} = \frac{e^{-3s}}{(s+1)^2(2s+1)},\ G_{p-reduction} = \frac{e^{-4.48s}}{2.95s+1}$$

Here, the simplified model was obtained by the model reduction technique by Chen [16]. 10% modeling errors both in the time delay and the time constant are assumed. Then the approximating transfer function is obtained as

$$G^*_m(s)K_{eq}\mathrm{e}^{-\theta_{eq}s} = \frac{\mathrm{e}^{-0.743s}}{2.66s+1}$$

Fig. 7 shows the closed loop responses of the conventional feedback system with the PID controller and the Smith predictor system with the PI controller. Both the PID controller in the conventional feedback system and the PI controller in the Smith predictor system are tuned by the IMC-PID tuning rule [12]. As can be seen, the performance of the Smith predictor system tuned by the proposed method is superior to that of the conventional controller. It illustrates the availability of the proposed method on the structural uncertainty of the model.

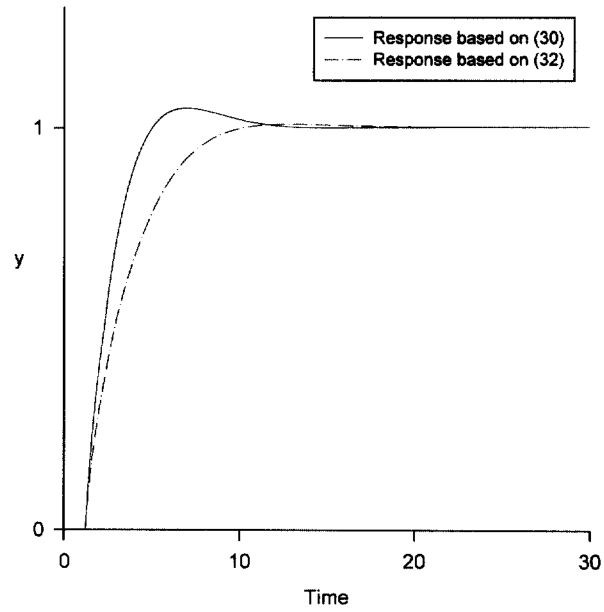

Fig. 6. Comparison of the closed loop responses for a unit step change in set-point (Example 2).

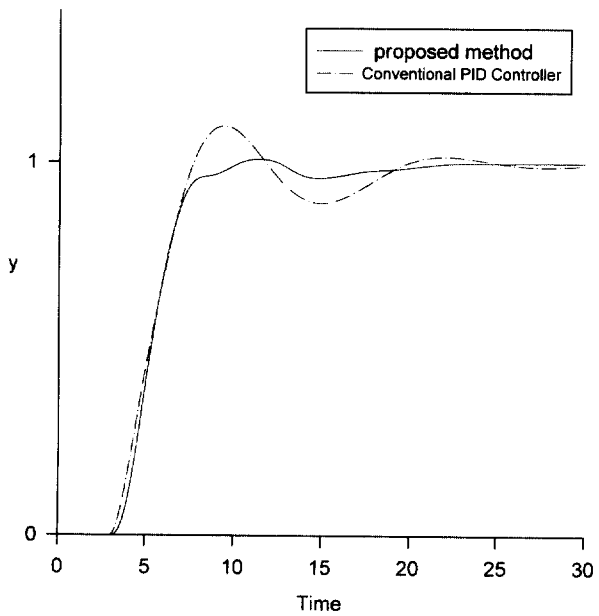

Fig. 7. Comparison of the closed loop responses for a unit step change in set-point (Example 3).

4. Conclusions

We propose the robust tuning method for the Smith predictor. The proposed method is based on the approximation by the equivalent gain plus time delay (EGPTD). It is shown that fairly good approximation over important frequency range could be achieved using the EGPTD. It is important that the proposed method can cope with simultaneous uncertainties in all the parameters of the model. Further, since the structure of the approximating model by the EGPTD remains same as that of the original model, many currently available PID tuning rules can be directly used for robust tuning of the Smith predictor system. Even though the proposed method is simple and easy, the robustness of the Smith predictor is ensured and more importantly, the tuning procedure is very simple and systematic.

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