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# Sequential loop closing identification of multivariable process models

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# Abstract

Sequential loop closing (SLC) method is one of the well-known methods to tune multiloop control systems for multivariable processes. In the method, each controller is designed sequentially with single-input single-output methods by finding the transfer function for the paired input and output while former loops have been closed. Utilizing the single-input single-output nature in tuning each controller, autotuning methods can also be applied. However, sometimes iterations are required for better performance. Especially, if pairing is undesirable, the multiloop control system designed with the autotuning SLC method does not show the best performance and tuning should be discarded and repeated totally for the correct pairing. Here, to avoid this, multivariable process models are identified while loops are being tuned. The identified models can be used to correct the pairing and to improve the multiloop control systems. Field experiments needed are just the same as the autotuning SLC method. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Identification of multivariable process models; Sequential loop closing method; Autotuning; Laguerre series method

# 1. Introduction

Multiloop control systems are often used in the chemical industry because of their simplicity. Many methods have been available to tune the multiloop control systems and still studied by many researchers. The sequential loop closing (SLC) method is one of the well-known methods to tune the multiloop control systems systematically (Mayne, 1973; Chiu & Arkun, 1992; Hovd & Skogestad, 1994). The method designs multiloop controllers sequentially. The first loop is designed for the first pair of inputs and outputs and it is closed. The second loop is designed while the first loop has been closed. Since the first loop is closed, the transfer function of the second pair is changed and hence design of the second loop should be done with the changed transfer function. In this manner, all loops are designed. Each controller is designed based on the transfer function between the paired input and output while former loops have been closed. One potential disadvantage is that the control performance is highly dependent on which loop is designed first and how it is designed. Specifically, the control performance of the multiloop control system which is designed with the SLC method can be worse if pair of inputs and outputs is undesirable or the design sequence is not appropriate.

Faster loops with higher ultimate frequencies are usually tuned first. Since a faster loop is less affected from slower loops, faster loops can be treated as decoupled loops and designed independently. On the other hand, when some loops are comparable speeds, the tuning sequence should be repeated for better control performance. To reduce effects of early loops on later loops, Chiu and Arkun (1992) used approximate transfer functions of later loops which have not been designed yet. Shen and Yu (1994) used a modified Ziegler-Nichols tuning rule for conservative tuning and repeated the design sequence once more. Shiu and Hwang (1998) decompose the multivariable system into several subsystems having comparable speeds. Tuning sequence is repeated within subsystems until iteration converges.

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The concept of SLC method is to design the multiloop control system in a sequence of single variable designs by identifying one transfer function between the paired input and output at each step. Hence we can use single-input single-output autotuning methods to design each loop (Shen & Yu, 1994; Semino & Scali, 1996; Loh, Hang, Quek & Vasnani, 1993). Autotuning SLC methods require minimal process information such as the input-output pairs and the design sequence. However, if pairing of inputs and outputs is undesirable and design sequence is not appropriate, tuning results should be discarded and tuning sequence should be repeated wholly for the correct pair and sequence, yielding long field tests. For some processes, undesirable pairing can be detected during the sequence of loop closing by checking the sign change at the steady state (Shen & Yu, 1994). However, the best pairing cannot be found during the sequence of loop closing in general. Here, to avoid these problems of autotuning SLC methods, we identify the whole multivariable process model instead of finding one transfer function at each step. Without any additional experimental load, the whole multivariable process model can be obtained while loops are tuned. The model can be used to correct the pairing, determine the loop closing sequence and tune multiloop control systems.

# 2. Sequential loop closing method

Consider a  $n \times n$  multivariable process whose transfer function matrix is

$$G(s) = \{g_{ij}(s), \quad i = 1, 2, ..., n, j = 1, 2, ..., n\}$$
(1)

The first m-1 loops are assumed closed already with a multiloop controller (Fig. 1)

$$C_1(s) = \text{diag}\{c_i(s), \quad i = 1, 2, \dots (m-1)\}$$
(2)

Then the transfer function between the mth pair becomes



Fig. 1. A partial control system for the SLC identification.

$$q_{\rm mm}(s) = g_{\rm mm}(s) - P_2(s)C_1(s)(I + P_1(s)C_1(s))^{-1}P_3(s)$$
(3)

where,

$$P_{1}(s) = \begin{bmatrix} g_{11}(s) & \cdots & g_{1,m-1}(s) \\ \cdots & \cdots & \cdots \\ g_{m-1,1}(s) & \cdots & g_{m-1,m-1}(s) \end{bmatrix}$$
$$P_{2}(s) = [g_{m1}(s) & \cdots & g_{m,m-1}(s)],$$
$$P_{3}(s) = \begin{bmatrix} g_{1,m}(s) \\ \vdots \\ g_{m-1,m}(s) \end{bmatrix}$$

The *m*th loop is designed for the transfer function (Eq. (3)). The transfer function  $q_{mm}(s)$  usually cannot be approximated well with the first order plus time delay model. Hence, to tune the *m*th loop, the frequency response methods such as the Ziegler-Nichols method and the gain and phase margins method are often used. Shen and Yu (1994) used the modified Ziegler-Nichols rule;

$$K_{\rm c} = \frac{K_{\rm u}}{3}$$

$$\tau_1 = 2P_{\rm u} \tag{4}$$

for the multiloop *PI* control system. Here  $K_c$  is the controller gain,  $\tau_1$  is the integral time, and  $K_u$  and  $P_u$  are the ultimate gain and ultimate period of  $q_{mm}(s)$ , respectively. The modification is to detune the controller for conservative responses.

Applying this procedure sequentially from the first controller to the last controller, we can design multiloop control systems.

#### 3. Sequential loop closing identification

In the SLC method, the transfer function of  $q_{mm}(s)$ under the former loops closed is identified at each step using single variable identifications. Instead, we identify the whole transfer function matrix while multiloop control system is tuned sequentially.

## 3.1. $2 \times 2$ Process identification

Consider a  $2 \times 2$  open-loop stable process;

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$
(5)

3.1.1. Step 1

We identify  $g_{11}(s)$  and  $g_{21}(s)$  by perturbing the input  $u_1$ . Any single variable identification method can be used. Here, the Laguerre filter method (Zervos et al., 1988; Park et al., 1997) is used. The method fits outputs with a truncated Laguerre series;

$$\hat{y}(t) = y_{ss} + \sum_{i=1}^{M} a_i \ell_i(t)$$
  
$$\ell_i(t) = \sqrt{2p} \frac{e^{-pt}}{(i-1)!} \frac{d^{i-1}(t^{i-1}e^{-2pt})}{dt^{i-1}}$$
(6)

where,  $\uparrow$  means an estimate, p is a time scale factor and  $a_i$ s are fitting parameters. Since the Laguerre coefficient  $a_i$ s appear linearly in Eq. (6), it can be identified by the standard least squares method with a fitting criterion,  $J = \sum_{m=1}^{N} (y(t_m) - \hat{y}(t_m))^2$ , where,  $t_m$  is time at the *m*th measurement. The identified model is

$$\hat{g}(s) = \frac{y_{ss}}{s} + \sum_{i=1}^{M} a_i L_i(s)$$

$$L_i(s) = \sqrt{2p} \frac{(s-p)^{i-1}}{(s+p)^i}$$
(7)

After identifying the Laguerre model with sufficiently larger M (here, M = 16 is used), we apply the model reduction method based on the balanced realization to it. By adjusting the time scale factor p a little we find a reduced model with minimal order.

Since the above Laguerre model is usually high order, sometimes it is inconvenient. However, the frequency response can be easily obtained. So we extract convenient low order parametric model from the frequency response. We use first order plus time delay (FOPTD) model which have many tuning rules for PID controllers;

$$f(s) = \frac{ke^{-\partial s}}{\tau s + 1} \tag{8}$$

whose parameters are

$$k = g(0)$$
  

$$\tau = \operatorname{avg}\left(\frac{\sqrt{(k/|g(j\omega|)^2 - 1}}{\omega}\right)$$
  

$$\theta = \operatorname{avg}\left(\frac{-\angle g(j\omega) - \tan^{-1}(\tau\omega)}{\omega}\right)$$
(9)

where, avg(.) means the average for a given frequency range.

3.1.2. Step 2

Tune the first controller  $c_1(s)$  for the transfer function  $\hat{g}_{11}(s)$ , the estimate of  $g_{11}(s)$ , and close the first loop. Then perturbation is introduced in the second input  $u_2$ . Transfer functions for the input  $u_2(s)$  are

$$u_{1}(s) = q_{12}(s)u_{2}(s) = -\frac{c_{1}(s)g_{12}(s)}{1 + g_{11}(s)c_{1}(s)}u_{2}(s)$$
  

$$y_{2}(s) = q_{22}(s)u_{2}(s) = \left(g_{22}(s) - \frac{g_{21}(s)c_{1}(s)g_{12}(s)}{1 + g_{11}(s)c_{1}(s)}\right)u_{2}(s)$$
(10)

By applying the above identification method to the output  $y_2$  and the input  $u_1$ , we can obtain models for  $q_{12}(s)$  and  $q_{22}(s)$ . Then, we have

$$\hat{g}_{12}(s) = \frac{-\hat{q}_{12}(s)(1+\hat{g}_{11}(s)c_1(s))}{C_1(s)}$$
$$\hat{g}_{22}(s) = \hat{q}_{22}(s) + \frac{\hat{g}_{21}(s)c_1(s)\hat{g}_{12}(s)}{1+\hat{g}_{11}(s)c_1(s)}$$
(11)

From the identified Laguerre models of  $\hat{g}_{12}(s)$  and  $\hat{g}_{22}(s)$ , we can obtain their frequency response data and, by fitting them, we can get approximate find first order plus time delay models.

#### 3.2. $n \times n$ Process identification

First, by perturbing the first input  $u_1$ , we can obtain  $g_{i1}(s)$ , i = 1, 2, ..., n as usual. From  $g_{11}(s)$ , the first controller  $c_1(s)$  is designed. When the first loop has been closed, we have

$$u_{1}(s) = q_{12}(s)u_{2}(s) = -\frac{c_{1}(s)g_{12}(s)}{1 + c_{1}(s)g_{11}(s)}u_{2}(s)$$
  

$$y_{i}(s) = q_{i2}(s)u_{2}(s) = \left(g_{i2}(s) - \frac{c_{1}(s)g_{i1}(s)g_{12}(s)}{1 + c_{1}(s)g_{11}(s)}\right)u_{2}(s),$$
  

$$i = 2, 3, ..., n$$
(12)

Hence, by perturbing the second input  $u_2$  under the first loop closed, we can obtain  $\hat{q}_{i2}(s)$ , i = 1, 2, ..., n and we have

$$\hat{g}_{12}(s) = -\hat{q}_{12}(s) \frac{1 + c_1(s)\hat{g}_{11}(s)}{c_1(s)}$$
$$\hat{g}_{i2}(s) = \hat{q}_{i2}(s) - \hat{g}_{i1}(s)\hat{q}_{12}(s), \quad i = 2, 3, ..., n$$
(13)

In general, assume that m-1 loops have been closed. Transfer functions for the *m*th input are

$$\begin{bmatrix} u_{1}(s) \\ \vdots \\ u_{m-1}(s) \end{bmatrix} \equiv Q_{3}(s)u_{m}(s)$$
  
=  $-C_{1}(s)(I + P_{5}(s)C_{1}(s))^{-1}P_{7}(s)u_{m}(s)$   
$$\begin{bmatrix} y_{m}(s) \\ \vdots \\ y_{n}(s) \end{bmatrix} \equiv Q_{4}(s)u_{m}(s) = (P_{8}(s) + P_{6}(s)Q_{3}(s))u_{m}(s)$$

(14)

where,

$$P_{5}(s) = \begin{bmatrix} g_{11}(s) & \cdots & g_{1,m-1}(s) \\ \cdots & \cdots & \cdots \\ g_{m-1,1}(s) & \cdots & g_{m-1,m-1}(s) \end{bmatrix},$$

$$P_{6}(s) = \begin{bmatrix} g_{m1}(s) & \cdots & g_{m,m-1}(s) \\ \cdots & \cdots & \cdots \\ g_{n,1}(s) & \cdots & g_{n,m-1}(s) \end{bmatrix},$$

$$P_{7}(s) = \begin{bmatrix} g_{1,m}(s) \\ \vdots \\ g_{m-1,m}(s) \end{bmatrix}, \quad P_{8}(s) = \begin{bmatrix} g_{m,m}(s) \\ \vdots \\ g_{n,m}(s) \end{bmatrix}$$

Hence we have

$$\hat{P}_{7}(s) = \begin{bmatrix} \hat{g}_{1,m}(s) \\ \vdots \\ \hat{g}_{m-1,m}(s) \end{bmatrix} = -(C_{1}^{-1}(s) + \hat{P}_{5}(s))\hat{Q}_{3}(s)$$

$$\hat{P}_{8}(s) = \begin{bmatrix} \hat{g}_{m,m}(s) \\ \vdots \\ \hat{g}_{n,m}(s) \end{bmatrix} = \hat{Q}_{4}(s) - \hat{P}_{6}(s)\hat{Q}_{3}(s)$$
(15)

In this manner, full multivariable process transfer functions can be determined while multiloop control system is being designed.

To identify  $Q_3(s)$  and  $Q_4(s)$ , we use the Laguerre filter method as above. With Eq. (15), we calculate  $\hat{g}_{im}(j\omega)$ , i=1, 2, ..., n, from  $\hat{Q}_3(j\omega)$  and  $\hat{Q}_4(j\omega)$ . Their computations in frequency domain are easy. From  $\hat{g}_{12}(s)$ , we extract the approximate first order plus time delay models for  $\hat{g}_{12}(s)$ .

#### 4. Usage of the identified model

Instead of identifying one transfer function at each step in autotuning SLC method (Shen & Yu, 1994; Loh et al., 1993), we identify full transfer function matrix. The multivariable process model can be used to improve the autotuning SLC method without iteratively applying the method to the real process, which requires long field experiments.

When the pairing is inadequate, good control performance cannot be expected for any multiloop control system including what is designed with the autotuning SLC method. For some multiloop control systems with pairing of negative relative gain array, the control system does not have loop failure tolerance so that the control system can be unstable when some loops are open or manipulated variables are on their limits. Besides the integrity problem, the control performances are usually very poor. The multivariable model identified while loops are being designed can be used to check and correct the pairing. The performance of multiloop control system designed with SLC methods is strongly dependent on which loop is designed first and how it is designed. To reduce this problem, Shen and Yu (1994) suggested some iterations. After all loops have been designed, the first loop is redesigned while all other loops have been closed. In this manner, loops up to n-1 loop are redesigned once more. If multivariable process models are available, this iteration can be done in the computer. It will reduce the design time and cost due to long field experiments considerably.

Loop failure tolerance is easily obtainable with the multiloop control systems. However, the SLC method does not guarantee it. Since some loops can be in manual mode for some reasons such as changing the control configuration and changing operation conditions, the loop failure tolerance is a very important feature in the industry. If a multivariable model is available, the loop failure tolerance can be checked although not exact because the identified model has uncertainties.

Identified models can be used to design control systems other than the multiloop control systems. Even a partial decoupling control system is used; a considerable improvement in control performance can be obtained for some interacting multivariable processes.

# 5. Case studies

Simulations are carried out to show that the proposed identification method provide models for multivariable processes and can be used to improve the autotuning SLC method.

**Example 1.** Consider the Wood and Berry (1984) column:

$$G_{\rm WB}(s) = \begin{bmatrix} \frac{12.8 \exp(-s)}{16.7s+1} & -\frac{18.9 \exp(-3s)}{21s+1} \\ \frac{6.6 \exp(-7s)}{10.9s+1} & -\frac{19.4 \exp(-3s)}{14.4s+1} \end{bmatrix}$$

First, with the usual step change of the first input, we identify  $\hat{g}_{11}(s)$  and  $\hat{g}_{21}(s)$ . Here, the outputs are corrupted with random noises between 0.1 and -0.1. From  $\hat{g}_{11}(s)$ , we design the first controller by the Ziegler-Nichols tuning rule as  $c_1(s) = 1.17(1 + 1/2.64s)$  and close the first control loop. Under the first loop closed, responses for the unit step change in  $u_2$  are shown in Fig. 2.

They are fitted with the truncated Laguerre series. From the Laguerre series models, we obtain first order plus time delay models of  $\hat{g}_{12}(s)$  and  $\hat{g}_{22}(s)$ . The whole transfer functions obtained are



Fig. 2. Typical responses of  $u_1$  and  $y_2$  and their Laguerre polynomial fittings for the step change of  $u_2$  while the first loop is closed in Example 1.



Fig. 3. Control responses for the step set point change in Example 1. (Solid line, multiloop control system tuned with the exact process; dotted line, multiloop control system tuned with an identified model; dashed line, partial decoupling control system).



Fig. 4. Control responses for the step set point change in Example 2. (Solid line, multiloop control system tuned by the autotuning SLC method without pairing correction; dotted line, multiloop control system tuned with an identified model and pairing correction).

$$\hat{G}_{WB}(s) = \begin{bmatrix} \frac{12.8 \exp(-1.21s)}{15.6s+1} & \frac{-18.9 \exp(-2.85s)}{18.2s+1} \\ \frac{6.60 \exp(-6.55s)}{10.9s+1} & \frac{-19.4 \exp(-2.40s)}{13.5s+1} \end{bmatrix}$$

With this model, control systems can be designed. Fig. 3 shows typical responses of multiloop control systems tuned by the modified Ziegler-Nichols method (Shen & Yu, 1994). Multiloop control systems designed with the exact process and the identified model have similar responses. If more elaborate tuning rules are used instead of the modified Ziegler-Nichols method, better control performances may be obtained. A partial decoupling control system with

$$C(s) = \begin{bmatrix} \frac{0.4525s + 0.0425}{s} & 0\\ 0 & -\frac{0.0833s + 0.0033}{s} \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0\\ -6.65/19.5 & 1 \end{bmatrix}$$

can be designed. Its response is also shown in Fig. 3. Even though a static decoupler is used, far better response is obtained.

**Example 2.** Consider again the Wood and Berry column;

$$G_2(s) = \begin{bmatrix} -\frac{18.9 \exp(-3s)}{21s+1} & \frac{12.8 \exp(-s)}{16.7s+1} \\ -\frac{19.4 \exp(-3s)}{14.4s+1} & \frac{6.6 \exp(-7s)}{10.9s+1} \end{bmatrix}$$

Here, the pairing is changed. Since the diagonal elements of the relative gain array of the steady-state gain matrix is negative ( $\lambda_{11} = -1.01$ ), this pairing suffers from lack of integrity when a multiloop control system with integral action is used. Applying the proposed identification method, we can obtain a model;

$$\hat{G}_{2}(s) = \begin{bmatrix} -\frac{18.9 \exp(-2.79s)}{21.0s+1} & \frac{12.8 \exp(-1.10s)}{16.0s+1} \\ -\frac{19.4 \exp(-3.03s)}{14.3s+1} & \frac{6.58 \exp(-4.92s)}{9.56s+1} \end{bmatrix}$$

In the SLC identification,  $c_1(s) = -0.293(1 + 1/8.59s)$  is used. From this model, we can correct the pairing and tune control systems. Multiloop control system with correct pairing is tuned by applying the SLC method. Responses for multiloop control system designed with the identified model are compared with

those designed with the autotuning SLC method of Shen and Yu (1994) in Fig. 4.

We can see that the autotuning SLC method shows very poor control performances. It is mainly because the pairing in the multiloop control system of the autotuning SLC method is undesirable. Actually it does not have loop failure tolerance, either. The autotuning SLC method should be repeated for the corrected pairing.

## 6. Conclusion

It is suggested to identify the full multivariable model while multiloop control system is tuned with the SLC method. The model can be used to correct the pairing, determine design sequence and design model based control systems. Field tests needed are the same as those for the autotuning SLC method. Simulations show that the proposed method, which tune the control system and identify the full multivariable model at the same time, is very useful to obtain better multiloop control systems.

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