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Mp Criterion Based Multiloop PID Controllers Tuning for Desired Closed Loop Responses

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Abstract−A tuning method of multiloop PID controllers is developed based on the generalized IMC PID tuning rule by Lee et al. (1998a). To extend the SISO PID tuning method to MIMO systems, a new tuning criterion is proposed. The criterion is based on the closed loop frequency response method to meet desired performance and robustness as close as possible. Examples for 2×2, 3×3 and 4×4 systems are used to illustrate the proposed method. The results show that the proposed method is superior to conventional methods such as the BLT tuning method.

Key words: Multiloop PID Controllers Tuning, Frequency Response, Mp

INTRODUCTION

Most chemical processes are basically MIMO systems. MIMO systems show special characteristics, namely, process interactions: each manipulated variable can affect all the controlled variables. The multiloop diagonal controller structure has been widely used for the multivariable processes because it usually provides quite adequate performance for process control applications while the structure is most simple, failure tolerant, and easy to understand [Grosdidier and Morari, 1987]. In order to solve the multiloop control problem, the best pairings of controlled and manipulated variables should first be determined by interaction analysis [Zhu and Jutan, 1996]. Once the control structure is fixed, the control performance is then mainly determined by the tuning of each multiloop PID controller.

Most multiloop PID controller tuning methods [Luyben, 1986; Grosdidier and Morari, 1987; Skogestad and Morari, 1989; Basualdo and Marchetti, 1990] currently available are similar in that they use the single loop tuning rules to obtain starting values for the individual controllers and then detune the individual loops to reserve stability of the overall system. For example, in the biggest log modulus tuning (BLT) method [Luyben, 1986], Zielgler-Nichols setting is used for initial settings for the individual controllers, then the controllers are detuned in such a way to satisfy the log modulus criterion. Economou and Morari [1986] developed the Internal Model Control (IMC) multiloop design method with the sufficient conditions for the stable filter to guarantee stability. However, it is known that the conditions are often too conservative and the resulting controllers give poor load disturbance response in certain situations [Ho et al., 1995].

In this paper, a new tuning method for the multiloop PID controllers is presented. The proposed method extends the SISO PID tuning method by Lee et al. [1998a] to the multiloop PID controllers. In order to consider the interaction effects by off-diagonal terms in an optimal sense, a criterion based on the closed loop frequency responses is presented to select a set of λ which corresponds to the closed loop time constant of the decentralized system. With this tuning criterion, the multiloop PID controllers can be designed to meet desired performance and robustness as close as possible.

The contents of the paper are arranged as follows. The extension of the generalized IMC-PID tuning method to MIMO system is given first. Next the Mp (peak magnitude ratio) tuning criterion to select a set of tuning parameters λ is presented. Simulation results using three representative examples for 2×2, 3×3 and 4×4 systems from the literature are given.

THEORY

1. Extension of the Generalized IMC-PID Tuning Method [Lee et al., 1998a] to MIMO System

Consider the n-inputs and n-outputs open loop stable multivariable process G with n-multiloop diagonal controllers G_c.

$$G_c(s)=\mathrm{diag}[G_{c1}, G_{c2}, \ldots, G_{cn}] \tag{1}$$

The multiloop feedback system can be generally represented by the block diagram in Fig. 1. The controller of the *i*th loop follows:

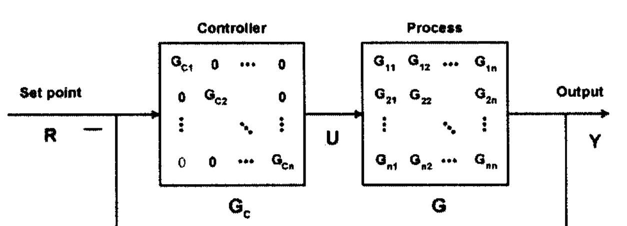

Fig. 1. Block diagram for multiloop control system.

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$$U_i = G_{ci}(Y_i - R_i) \tag{2}$$

Here, the inputs, outputs and set points are represented by $U=(U_1, U_2, \ldots, U_n)$, $Y=(Y_1, Y_2, \ldots, Y_n)$ and $R=(R_1, R_2, \ldots, R_n)$, respectively. One of the simple ways to extend the IMC structure for SISO system to the multiloop structure might consider a transfer function matrix which includes only the diagonal elements of G(s).

$$\tilde{G}(s) = \mathrm{diag}[G_{11}, G_{22}, \ldots, G_{nn}] \tag{3}$$

The multiloop controllers in classical feedback structure are then represented by

$$G_c(s) = [I - Q(s)\tilde{G}(s)]^{-1} Q(s) \tag{4}$$

where Q is a diagonal matrix representing the IMC controller. Because all the matrix are diagonal forms in Eq. (4), each controller in a loop can be designed as follows:

Let us consider the controller in the ith loop. Generally, a stable process model of the ith row and the ith column of G matrix can be represented as follows:

$$G_{ii}(s) = P_{ii}^m(s) P_{ii}^A(s) \tag{5}$$

where $P_{ii}^m(s)$ is the portion of the model inverted by the controller (it must be a minimum phase), $P_{ii}^A(s)$ is the portion of the model not inverted by the controller (it is usually a non-minimum phase, that is, it contains dead times and/or right half plane zeros).

To give the best lest-squares response, the portion of the model not inverted by the controller is chosen to be the all pass form as

$$P_{ii}^A(s) = \prod_{j,k} \left(\frac{-\tau_j s + 1}{\tau_j s + 1} \right) \left(\frac{\tau_k^2 s^2 - 2\tau_k \zeta_k s + 1}{\tau_k^2 s^2 + 2\tau_k \zeta_k s + 1} \right) e^{-T_{ii} s} \tag{6}$$

The requirement of $P_{ii}^A(0)=1$ is necessary for the controlled variable to track its set point because this adds integral action to the controller. Here, our purpose is to design the controller G_{ci} to make the closed loop transfer function to follow a desired closed loop response of loop i given by

$$\frac{C}{R_i} = \frac{P_{ii}^A(s)}{(\lambda_i s + 1)^{r_i}} \tag{7}$$

The term $1/(\lambda_i s+1)^{r_i}$ is an IMC filter with an adjustable time constant of the ith loop, then G_{ci} that gives the desired loop response is given by

$$G_{ci}(s) = \frac{Q_i}{1 - G_{ii} Q_i} = \frac{(P_{ii}^m(s))^{-1}}{(\lambda_i s + 1)^{r_i} - P_{ii}^A(s)} \tag{8}$$

where Q_i is the IMC controller represented by $(P_{ii}^m(s))^{-1}/(\lambda_i s+1)^{r_i}$. Since $P_{ii}^A(0)$ is 1, the controller G_{ci} can be expressed with an integral term as

$$G_{ci} = f(s)/s \tag{9}$$

In order to approximate the above ideal controller to a PID controller, expanding G_{ci} in a Maclaurin series in s gives

$$G_{ci}(s) = \frac{1}{s}\left(f(0) + f'(0)s + \frac{f''(0)}{2} s^2 + \cdots \right) \tag{10}$$

The resulting controller includes an infinite number of high-order s terms. Among them, the first three terms correspond to the integral term, the proportional term, and the derivative term of the PID controller, respectively. Since the controller given by Eq. (10) is equivalent to the controller given by Eq. (8), the desired closed loop response can be perfectly achieved when all terms in Eq. (10) are considered. However, in practice, it is impossible to consider the infinite number of high-order derivative terms in the controller given by Eq. (10).

Since in the actual control situation low and middle frequencies are much more important than high frequencies, and only the first three terms in Eq. (10) are often sufficient to achieve the desired closed loop performance, the controller given by Eq. (10) can be approximated to the PID controller by using only the first three terms (1/s, 1, s) in Eq. (10) and truncating all other high-order terms (s^2, s^3, …). As a result, the controller can be approximated with the first three terms as the standard PID controller given by

$$G_{ci} = K_{ci}\left(1 + \frac{1}{\tau_{Ii} s} + \tau_{Di} s \right) \tag{11}$$

where

$$K_{ci} = f'(0); \quad \tau_{Ii} = f'(0)/f(0); \quad \tau_{Di} = f''(0)/2f'(0) \tag{12}$$

The above formulas are used to obtain the tuning rules as analytical functions of the process model parameters and the closed loop time constants λ. In the specific case where the process model has strong lead term, the derivative and/or integral time constants computed from Eq. (11) can be negative values independent of the choice of the closed loop time constant. In that case, a PID controller cascaded with a first or second order lag of the form $1/(\alpha s+1)$ or $1/(\alpha_2 s^2 + \alpha_1 s+1)$ is recommended [Lee et al., 1998a]. To obtain a PID controller cascaded by a first order lag that is $K_{ci}(1+1/\tau_{Ii}s+\tau_{Di}s)/(\alpha s+1)$, we rewrite G_{ci} as

$$G_{ci}(s) = \frac{1}{s} f(s) = \frac{1}{s} \frac{f(s)h(s)}{h(s)} \tag{13}$$

where $h(s)=1+\alpha s$.

Now, we expand the quantity f(s)h(s) in a Maclaurin series about the origin and choose the parameter α so that the third order term in the expansion becomes zero. The expansion of Eq. (13) then becomes

$$G_{ci}(s) = \{f(0) + [f'(0) + \alpha f(0)]s + [f''(0) + 2\alpha f'(0)]s^2/2 + [f'''(0) + 3\alpha f''(0)]s^3/6 + \cdots\}/s(\alpha s+1) \tag{14}$$

and the PID parameters are

$$K_{ci} = f'(0) + \alpha f(0); \quad \tau_{Ii} = K_{ci}/f(0);$$
$$\tau_{Di} = [f''(0) + 2\alpha f'(0)]/(2K_{ci}); \quad \alpha = -f'''(0)/3f''(0) \tag{15}$$

By Eqs. (12) and (15), all the controllers in G_c can be designed for general process models. PID tuning values can be chosen by selecting λ, here, $\lambda=(\lambda_1, \lambda_2, \ldots, \lambda_n)$. As the λ_i decreases, the closed loop response of loop i becomes faster or even unstable. On the other hand, as the λ_i increases, the closed loop response of loop i becomes more sluggish or stable. Therefore, an appropriate set of λ to compromise stability and performance in MIMO system has to be chosen.

2. Mp Tuning Criterion for the Multiloop Controllers Tuning

The criterion considered in this paper is based on the closed loop frequency response which obtains from the calculation of the system output in response to a sinusoidal input. The use of frequency response has the following advantages [Seborg et al., 1989]:

• It is applicable to dynamic models of any order.
• The designer can specify the desirable closed loop response characteristics.
• Information on stability margins and sensitivity characteristics is provided.

In MIMO systems, the closed loop transfer function can be represented by

$$H(s)=G(s)G_c(s)[I+G(s)G_c(s)]^{-1} \tag{16}$$

The closed loop transfer function can be expressed by the following matrix form

$$H(s)=\begin{bmatrix} H_{11}\ H_{12}\cdots H_{1n} \\ H_{21}\ H_{22} & \\ \vdots & \ddots\ \vdots \\ H_{n1} & \cdots H_{nn}\end{bmatrix} \tag{17}$$

or

$$H=\{H_{ij}\},\ i=1, \ldots, n, j=1, \ldots, n \tag{18}$$

where H_{ij} represents the closed loop response of the ith loop to the set-point change in the jth loop.

The closed loop frequency response can be found by setting $s=j\omega$ in Eq. (16) and can be represented in terms of ω and λ as follows:

$$H(j\omega, \lambda)=G(j\omega)G_c(j\omega, \lambda)[I+G(j\omega)G_c(j\omega, \lambda)]^{-1} \tag{19}$$

The magnitude of frequency response is called the closed loop amplitude ratio (AR) and its maximum magnitude over the frequency range is defined as Mp. The AR curve of diagonal elements should be unity to as high a frequency as possible to ensure no offset and a rapid approach to the new steady state, while that of off-diagonal elements should be as small as possible [Edgar et al., 1981]. Mp is closely related to the stability and performance of the closed loop system, and the corresponding response in the time domain can be inferred from the value of Mp.

Here, our aim is to find a set of λ to make closed loop response fast and stable enough. The above time domain objective can be achieved by solving the following optimization problem in the frequency domain:

$$\min_{\lambda_i}\left[\sum_i\sum_{j\neq i}Mp_{ij}+w\sum_i Mp_{ii}\right] \tag{20}$$

s.t. $Mp_{ii} \geq Mp_{low}$

where $Mp_{ij}=\max_\omega H_{ij}$; Mp_{ij} is the function of λ; Mp_{low} is the lower bound of diagonal Mp; w is the weighting factor for the diagonal Mp.

Minimizing the objective function as shown in Eq. (20) makes the process response stable and the constraints make process response fast enough. The constraint limits are predetermined to guarantee a required minimum speed of process responses. Usually, a value between 1.1 and 1.4 is selected as the lower bound of diagonal Mp [Harris and Mellichamp, 1985]. Since the optimization problem consists of only one optimizing variable vector, one can easily find the optimum λ to minimize interaction and overshoot in the loops while all Mp_{ii} meet the constraint limits.

SIMULATION RESULTS

Simulation studies are carried out to illustrate the proposed tuning method. We discuss our proposed tuning rules with 2×2 case, 3×3 case and 4×4 case from the open literatures. The results are compared with those by the BLT method.

1. Example 1 (2×2 System)

First, the Wood and Berry (WB) column model [Luyben, 1986] was studied.

Table 1. Optimization results of example 1 for various parameter sets in the proposed criterion

Mp_{low}	w	Loop 1			Loop 2		
		λ	ω_p	Mp	λ	ω_p	Mp
1.05	0	0.0708	2.31	1.62	3.01	0.464	1.05
	1	0.288	0.433	1.18	3.09	1.464	1.05
	1.5	0.302	0.433	1.184	3.09	0.464	1.05
1.1	0	0.0647	2.31	1.65	2.84	0.500	1.1
	1	0.276	0.464	1.20	2.91	0.464	1.1
	1.5	0.239	0.464	1.19	2.89	0.464	1.1
1.15	0	0.0633	2.31	1.66	2.71	0.498	1.15
	1	0.259	0.464	1.20	2.73	0.464	1.15
	1.5	0.240	0.464	1.20	2.72	0.464	1.15

Table 2. Tuning results by the proposed method and the BLT method for three examples

		BLT method	Proposed method
WB	λ	-	0.276, 2.91
	K_c	0.375, −0.075	1.047, −0.132
	τ_I	8.29, 23.6	17.1, 15.2
	τ_D	-	0.374, 0.712
	IAE1*	5.11, 16.8	3.42, 6.04
	IAE2	3.38, 32.7	2.46, 12.1
T4	λ	-	0.114, 0.214, 1.63
	K_c	−11.26, −3.52, −0.182	−40.9, −13.5, −0.372
	τ_I	7.09, 14.5, 15.1	67.0, 4.93, 11.8
	τ_D	-	0.305, 1.31, 0.375
	IAE1	10.3, 9.71, 35.7	2.24, 1.57, 6.11
	IAE2	25.8, 40.7, 148.6	3.95, 6.89, 19.6
	IAE3	9.02, 12.6, 48.1	1.81, 2.66, 10.9
A1	λ	-	0.629, 0.377, 0.0617, 0.894
	K_c	2.28, 2.94, 1.18, 2.02	7.40, 6.75, 3.53, 3.77
	τ_I	72.2, 7.48, 7.39, 27.8	61.7, 32.4, 16.7, 53.0
	τ_D	-	15.3, 0.364, 0.467, 4.48
	IAE1	60.8, 4.27, 81.8, 74.8	22.2, 1.97, 14.7, 32.8
	IAE2	67.3, 8.58, 144, 118	12.5, 4.44, 65.9, 37.6
	IAE3	3.47, 0.134, 4.17, 3.20	0.957, 0.180, 2.74, 3.30
	IAE4	16.0, 2.28, 41.9, 36.0	1.57, 1.02, 9.24, 12.5

*IAE1: IAE for unit step change in loop 1.

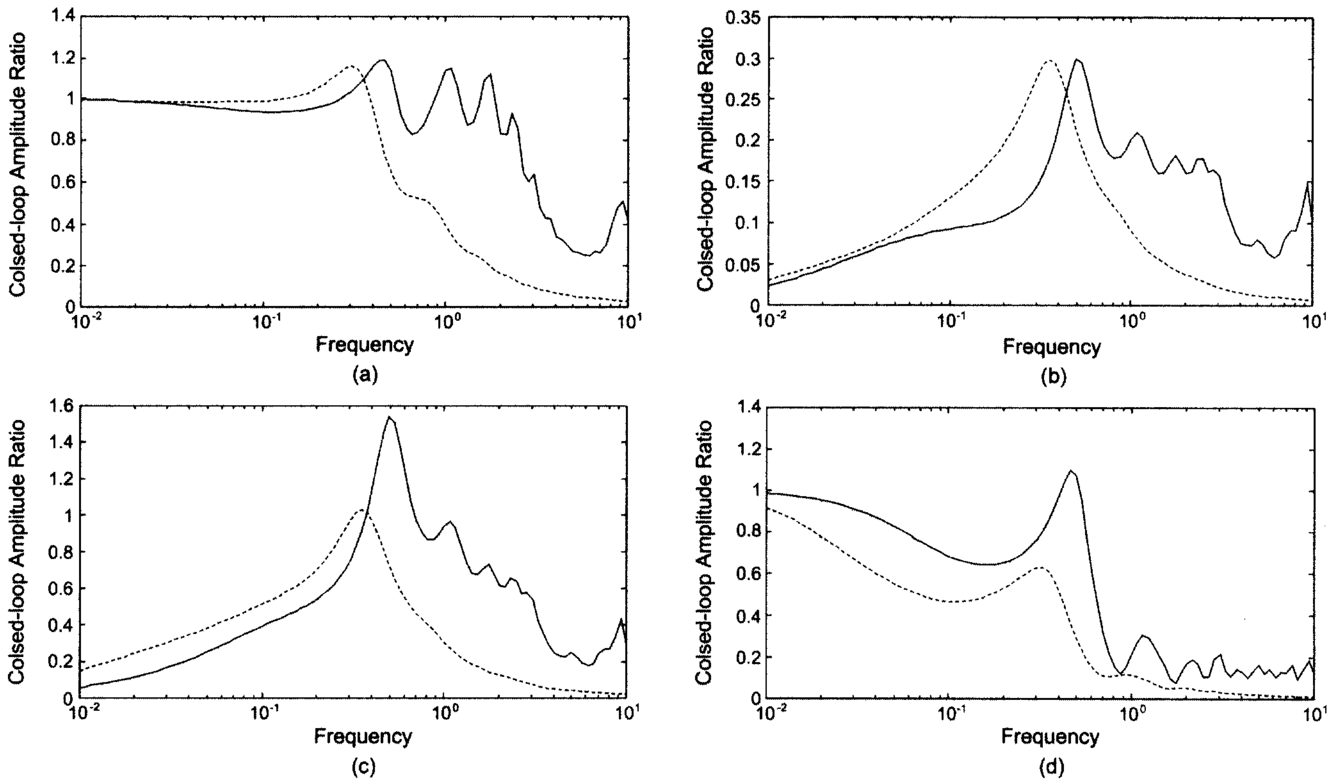

Fig. 2. Closed loop frequency responses for example 1 (WB). (a) H_{11} (b) H_{12} (c) H_{21} (d) H_{22} (Solid line: proposed method, dashed line: BLT method).

$$G(s)=\begin{bmatrix}\dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1}\\[2mm] \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1}\end{bmatrix} \tag{21}$$

The PID controllers of loop 1 and loop 2 for the WB model were tuned by the proposed tuning rule. Table 1 shows the results for various sets of Mp_{low} and w. In this study, Mp_{low} and w are set by 1.1 and 1, respectively, for all the examples. The resulting tuning parameters are listed in Table 2. Figs. 2 and 3 show the closed loop responses tuned by the proposed method and the BLT method both in the frequency and time domains. In addition, the performances of the final controller are evaluated via the IAE values of all loops for a unit step change in each set point. The results show that the proposed method is better than the BLT method.

2. Example 2 (3×3 System)

Tyreus case 4 (T4) [Luyben, 1986] as the 3×3 system example was studied.

$$G(s)=\begin{bmatrix}\dfrac{-1.986e^{-0.71s}}{66.67s+1} & \dfrac{5.24e^{-60s}}{400s+1} & \dfrac{5.984e^{-2.24s}}{14.29s+1}\\[2mm] \dfrac{0.0204e^{-0.59s}}{(7.14s+1)^2} & \dfrac{-0.33e^{-0.68s}}{(2.38s+1)^2} & \dfrac{2.38e^{-0.42s}}{(1.43s+1)^2}\\[2mm] \dfrac{0.374e^{-7.75s}}{22.22s+1} & \dfrac{-11.3e^{-3.79s}}{(21.74s+1)^2} & \dfrac{-9.811e^{-1.59s}}{11.36s+1}\end{bmatrix} \tag{22}$$

The resulting tuning parameters are listed in Table 2. The same conclusion can be drawn as that from the above 2×2 example.

3. Example 3 (4×4 System)

Alatiqi case 1 (A1) [Luyben, 1986] as the 4×4 system example was studied.

$$G(s)=\begin{bmatrix}\dfrac{4.09e^{-1.3s}}{(33s+1)(8.3s+1)} & \dfrac{-6.36e^{-0.2s}}{(31.6s+1)(20s+1)} & \dfrac{-0.25e^{-0.4s}}{21s+1} & \dfrac{-0.49e^{-5s}}{(22s+1)^2}\\[2mm] \dfrac{-4.17e^{-4s}}{45s+1} & \dfrac{6.93e^{-1.01s}}{44.6s+1} & \dfrac{-0.05e^{-5s}}{(34.5s+1)^2} & \dfrac{1.53e^{-2.8s}}{48s+1}\\[2mm] \dfrac{-1.73e^{-17s}}{(13s+1)^2} & \dfrac{5.11e^{-11s}}{(13.3s+1)^2} & \dfrac{4.61e^{-1.02s}}{18.5s+1} & \dfrac{-5.48e^{-0.5s}}{15s+1}\\[2mm] \dfrac{-11.18e^{-2.6s}}{(43s+1)(6.5s+1)} & \dfrac{14.04e^{-0.02s}}{(45s+1)(10s+1)} & \dfrac{-0.1e^{-0.05s}}{(31.6s+1)(5s+1)} & \dfrac{4.49e^{-0.6s}}{(48s+1)(6.3s+1)}\end{bmatrix} \tag{23}$$

The resulting tuning parameters are listed in Table 2. The closed loop responses for this system are shown in Fig. 4. The tuning parameters by the proposed method give better closed loop responses than the BLT method. It can be seen from the figure that the proposed method reduces loop interactions considerably compared with the BLT method.

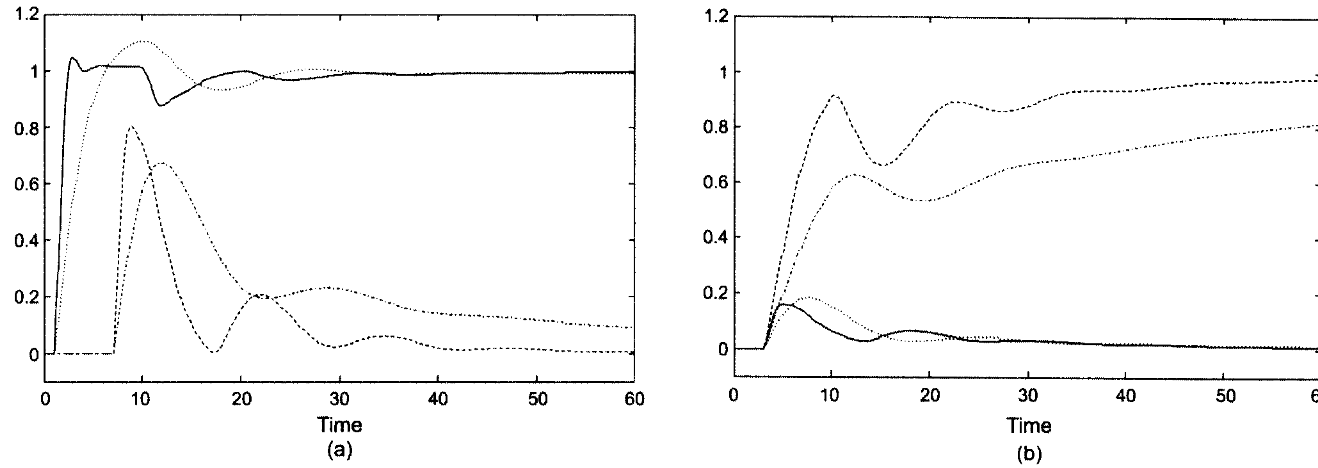

Fig. 3. Closed loop time response of each loop for example 1 (WB). (a) Set point change in loop 1; (b) Set point change in loop 2; (Solid line: loop 1 (proposed), dotted line: loop 1 (BLT), dashed line: loop 2 (proposed), dash dotted line: loop 2 (BLT)).

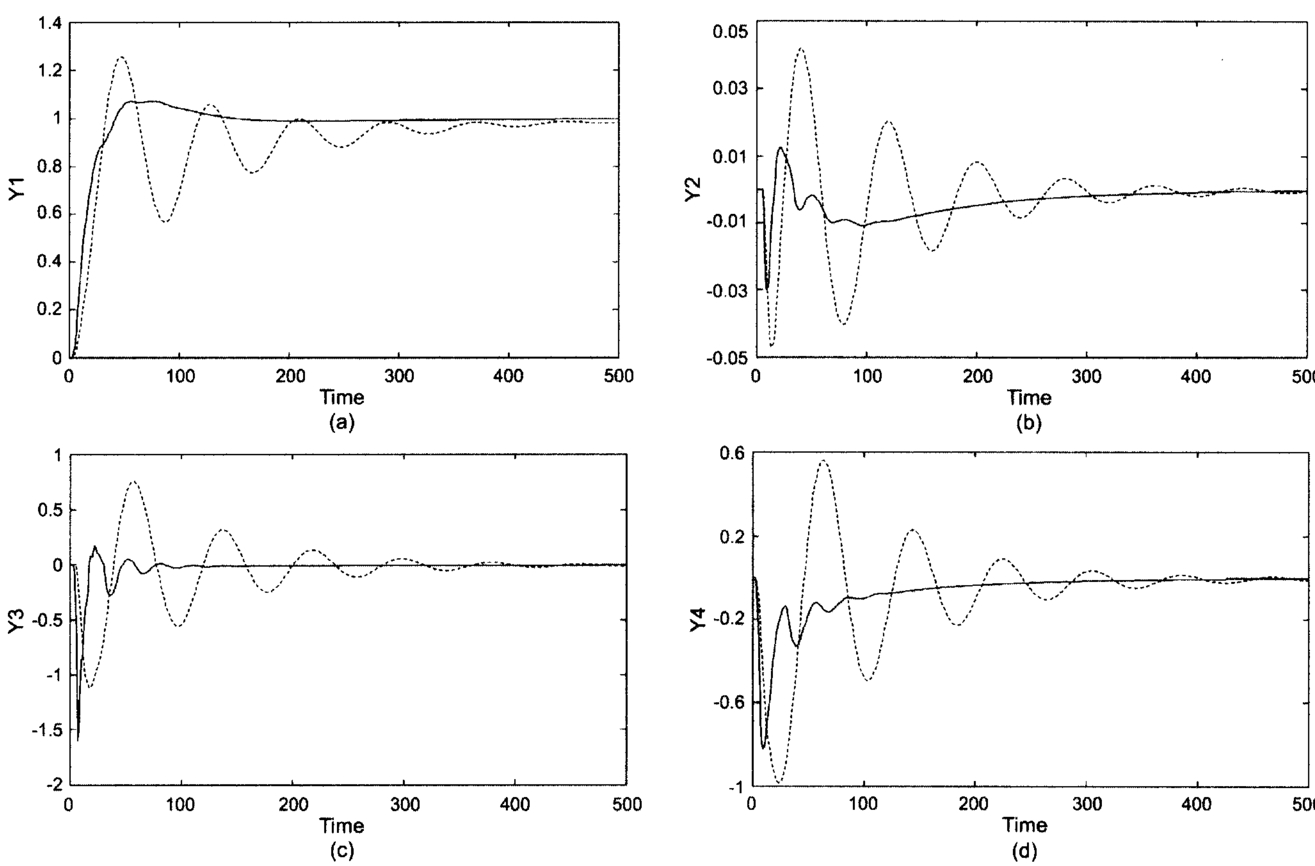

Fig. 4. Closed loop responses to unit step change in loop 1 set point for example 3 (A1). (a) Loop 1 (b) Loop 2 (c) Loop 3 (d) Loop 4 (Solid line: proposed method, dashed line: BLT method).

CONCLUSION

A tuning method for multiloop PID controllers is proposed by the extension of the generalized IMC PID tuning method for the SISO system [Lee et al., 1998a]. To cope with the interaction effects due to the off-diagonal elements, the optimum value of λ for each loop is found by solving a simple optimization problem based on Mp criterion. The resulting multiloop PID controllers make the closed loop responses meet desired performance and robustness as close as possible. This method shows good performance and stability compared with the existing method.

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