Analytical Design of Multiloop PID Controllers for Desired Closed-Loop Responses

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Most chemical processes are basically multiple input/multiple output (MIMO) systems. Despite considerable work on advanced multivariable controllers for MIMO systems, multiloop proportional-integral-derivative (PID) controllers remain the standard for many industries because of their adequate performance with most simple, failure tolerant, and easy to understand structure. In a multiloop system, once a control structure is fixed, control performance is then determined mainly by tuning each multiple single-loop PID controller. However, because the interactions that exist between the control loops make the proper tuning of the multiloop PID controllers quite difficult, only a relatively few tuning methods are available to the multiloop PID controllers and most of them require nonanalytical forms with complex iterative steps (Loh et al., 1993; Luyben, 1986; Skogestad and Morari, 1989). The analytical tuning rule is very attractive, with respect to its practicality, but the mathematical complexity attributed to the loop interactions has mainly prevented the analytical approach to the multiloop systems.

In this article, we propose an analytical design method for the multiloop PID controllers to give desired closed-loop responses by extending the generalized IMC–PID method for single input/single output (SISO) systems (Lee et al., 1998) to MIMO systems. Simple but efficient tuning rules are obtained for general process models by using the frequency-dependent property of the closed-loop interactions.

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Theory

Direct extension of generalized IMC-PID method to multiloop systems and its limitation

In the multiloop feedback system, shown in Figure 1, the closed-loop transfer function matrix $\mathbf{H}(s)$ is given by

$$\mathbf{y}(s) = \mathbf{H}(s)\mathbf{r}(s) = (\mathbf{I} + \mathbf{G}(s)\tilde{\mathbf{K}}(s))^{-1}\mathbf{G}(s)\tilde{\mathbf{K}}(s)\mathbf{r}(s) \tag{1}$$

where $\mathbf{G}(s)$ is the open-loop stable process, $\tilde{\mathbf{K}}(s)$ is the multiloop controller, and $\mathbf{y}(s)$ and $\mathbf{r}(s)$ are the controlled variable vector and the set-point vector, respectively.

According to the design strategy of the multiloop IMC controller (Economou and Morari, 1986), the desired closed-loop response R_i of the ith loop is typically chosen by

$$\frac{y_i}{r_i} = R_i = \frac{G_{ii+}(s)}{(\lambda_i s + 1)^{n_i}} \tag{2}$$

where G_{ii+} is the nonminimum part of G_{ii} and is chosen to be the all-pass form, λ_i is an adjustable constant for system performance and stability, and n_i is chosen such that the IMC controller would be realizable. The requirement of $G_{ii+}(0) = 1$ is necessary for the controlled variable to track its set point.

Let the desired closed-loop response matrix $\tilde{\mathbf{R}}(s)$ be

$$\tilde{\mathbf{R}}(s) = \mathrm{diag}[R_1, R_2, \ldots, R_n] \tag{3}$$

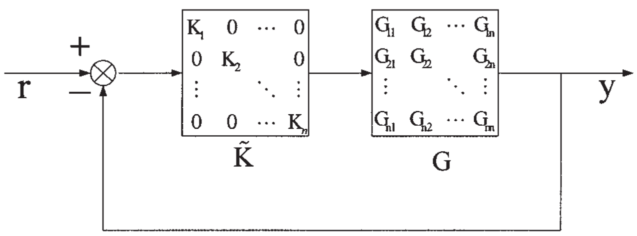

Figure 1. Block diagram for multiloop control system.

Then, our aim is to design the multiloop controller $\tilde{\mathbf{K}}(s)$ such that all the diagonal elements of $\mathbf{H}(s)$ resemble those of $\tilde{\mathbf{R}}(s)$ as close as possible over a frequency range relevant to control applications.

$\tilde{\mathbf{K}}(s)$, with an integral term, can be written in a Maclaurin series as

$$\tilde{\mathbf{K}}(s) = \frac{1}{s}[\tilde{\mathbf{K}}_0 + \tilde{\mathbf{K}}_1 s + \tilde{\mathbf{K}}_2 s^2 + O(s^3)] \tag{4}$$

Note that $\tilde{\mathbf{K}}_0$, $\tilde{\mathbf{K}}_1$, and $\tilde{\mathbf{K}}_2$ correspond to the integral, proportional, and derivative terms of the multiloop PID controller, respectively.

Expanding $\mathbf{G}(s)$ in a Maclaurin series also gives

$$\mathbf{G}(s) = \mathbf{G}_0 + \mathbf{G}_1 s + \mathbf{G}_2 s^2 + O(s^3) \tag{5}$$

where $\mathbf{G}_0 = \mathbf{G}(0)$, $\mathbf{G}_1 = \mathbf{G}'(0)$, and $\mathbf{G}_2 = \mathbf{G}''(0)/2$.

By substituting Eqs. 4 and 5 into Eq. 1 and rearranging it, we obtain $\mathbf{H}(s)$ in a Maclaurin series as

$$\begin{aligned}\mathbf{H}(s) = \mathbf{I} - (\mathbf{G}_0\tilde{\mathbf{K}}_0)^{-1}s + (\mathbf{G}_0\tilde{\mathbf{K}}_0)^{-1}(\mathbf{I} + \mathbf{G}_0\tilde{\mathbf{K}}_1 + \mathbf{G}_1\tilde{\mathbf{K}}_0) \\ (\mathbf{G}_0\tilde{\mathbf{K}}_0)^{-1}s^2 + (\mathbf{G}_0\tilde{\mathbf{K}}_0)^{-1}[\mathbf{G}_0\tilde{\mathbf{K}}_2 + \mathbf{G}_1\tilde{\mathbf{K}}_1 + \mathbf{G}_2\tilde{\mathbf{K}}_0 \\ - (\mathbf{I} + \mathbf{G}_0\tilde{\mathbf{K}}_1 + \mathbf{G}_1\tilde{\mathbf{K}}_0)(\mathbf{G}_0\tilde{\mathbf{K}}_0)^{-1} \\ (\mathbf{I} + \mathbf{G}_0\tilde{\mathbf{K}}_1 + \mathbf{G}_1\tilde{\mathbf{K}}_0)(\mathbf{G}_0\tilde{\mathbf{K}}_0)^{-1}]s^3 + O(s^4)\end{aligned} \tag{6}$$

$\tilde{\mathbf{R}}(s)$ can also be expressed in a Maclaurin series as

$$\tilde{\mathbf{R}}(s) = \tilde{\mathbf{R}}(0) + \tilde{\mathbf{R}}'(0)s + \frac{\tilde{\mathbf{R}}''(0)}{2}s^2 + \frac{\tilde{\mathbf{R}}'''(0)}{6}s^3 + O(s^4) \tag{7}$$

where $\tilde{\mathbf{R}}(0) = \mathbf{I}$, given that $G_{ii+}(0) = 1$.

By comparing each diagonal element of $\mathbf{H}(s)$ and $\tilde{\mathbf{R}}(s)$ in Eqs. 6 and 7 for the first three s terms (s, s^2, s^3), we can express $\tilde{\mathbf{K}}_0$, $\tilde{\mathbf{K}}_1$, and $\tilde{\mathbf{K}}_2$ in terms of the process model parameters and the desired closed-loop response parameters. Although the analytical tuning rules so obtained take the interaction effect of the multiloop system fully into account, some of them (that is, for $\tilde{\mathbf{K}}_1$ and $\tilde{\mathbf{K}}_2$) would be too complicated to use it practically and also often show severe inaccuracy because of the inherent limitation of a Maclaurin series for the high-frequency region. However, by using the frequency-dependent characteristics of the closed-loop interactions, these limitations can be successfully avoided and the simple but efficient tuning rules are available.

Design of proportional gain K_c and derivative time constant τ_D

As indicated from Eq. 4, the impact of $\tilde{\mathbf{K}}_1$ and $\tilde{\mathbf{K}}_2$ on the PID algorithm becomes more predominant at high frequencies,

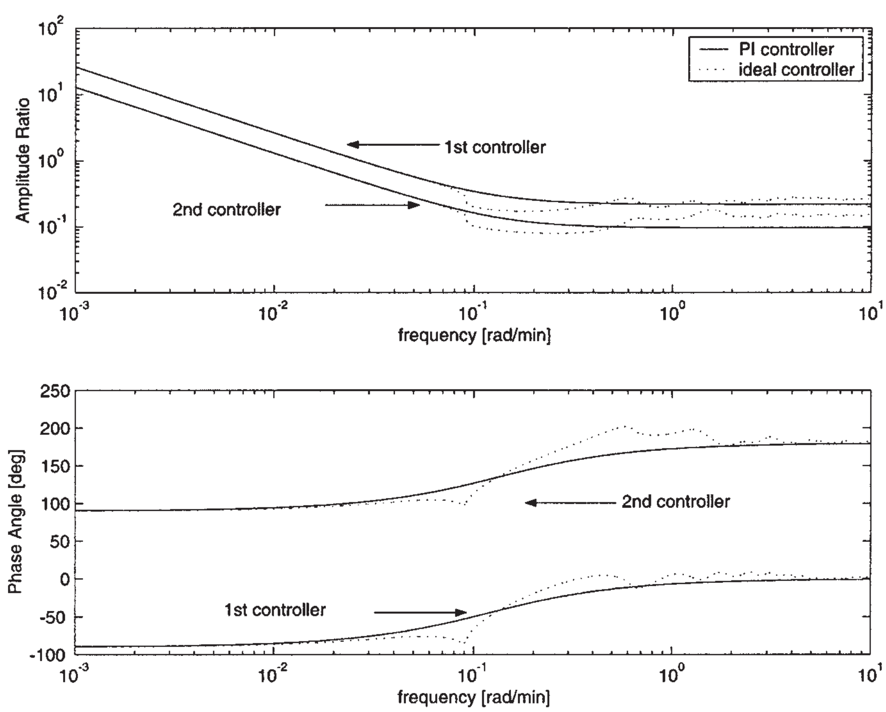

Figure 2. Bode diagrams for the ideal controllers and the proposed PI controllers for Wood and Berry column.

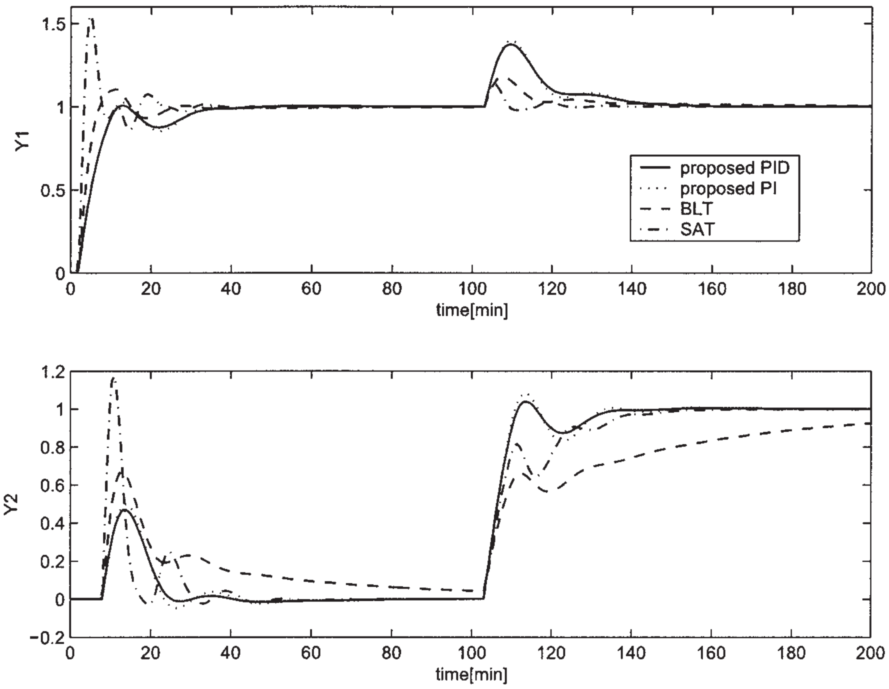

Figure 3. Closed-loop responses to sequential step changes in set point for Wood and Berry column.

whereas it is relatively insignificant at low frequencies. Therefore it is desirable for $\tilde{\mathbf{K}}_1$ and $\tilde{\mathbf{K}}_2$ to be designed based on the process characteristics at high frequencies.

Given that $|\mathbf{G}(jw)\tilde{\mathbf{K}}(jw)| \ll 1$ at high frequencies, $\mathbf{H}(s)$ can be approximated to

$$\mathbf{H}(s) = (\mathbf{I} + \mathbf{G}(s)\tilde{\mathbf{K}}(s))^{-1}\mathbf{G}(s)\tilde{\mathbf{K}}(s) \approx \mathbf{G}(s)\tilde{\mathbf{K}}(s) \tag{8}$$

This feature at high frequencies indicates that $\tilde{\mathbf{K}}_1$ and $\tilde{\mathbf{K}}_2$ can be designed by considering only the main diagonal elements in $\mathbf{G}(s)$. In fact, at high frequencies, the transmission interaction (that is, a change in a loop affects its controlled variable again through the other loops) is mostly eliminated by low-pass filtering through the processes in the other loops. Therefore, the generalized IMC-PID method for the SISO system (Lee et al., 1998) can be directly applied to the design of K_{ci} and τ_{Di} of the multiloop PID controller as follows.

Dropping the off-diagonal terms in $\mathbf{G}(s)$, we can obtain the ideal multiloop controller $\tilde{\mathbf{K}}(s)$ to give the desired closed-loop responses $\tilde{\mathbf{R}}(s)$ as

$$\tilde{\mathbf{K}}(s) = \tilde{\mathbf{G}}^{-1}(s)\tilde{\mathbf{R}}(s)[\mathbf{I} - \tilde{\mathbf{R}}(s)]^{-1} \tag{9}$$

where $\tilde{\mathbf{G}}(s) = \mathrm{diag}[G_{11}, G_{22}, \ldots, G_{nn}]$.

Thus, we can design the ideal controller $K_i(s)$ of the ith loop simply as

$$K_i(s) = \frac{Q_i(s)}{1 - G_{ii}(s)Q_i(s)} = \frac{[G_{ii-}(s)]^{-1}}{(\lambda_i s + 1)^{n_i} - G_{ii+}(s)} \tag{10}$$

where $Q_i(s)$ is the IMC controller given by $[G_{ii-}(s)]^{-1}/(\lambda_i S + 1)^{n_i}$.

Because $G_{ii+}(0)$ is 1, Eq. 10 can be rewritten in a Maclaurin series with an integral term as

Table 1. Tuning Results by the Various Methods in the Examples

Process	Tuning Method	Controller Parameter		
		K_{ci}	τ_{Ii}	τ_{Di}
WB column	Proposed PID*	0.219, −0.0964	8.35, 7.45	0.0817, 0.525
	Proposed PI*	0.219, −0.0964	8.35, 7.45	—
	BLT	0.375, −0.0750	8.29, 23.6	—
	SAT	0.868, −0.0868	3.25, 10.4	—
OR column	Proposed PI**	0.593, −0.124, 3.22	3.43, 2.88, 7.65	—
	BLT	1.510, −0.295, 2.63	16.4, 18.0, 6.61	—
	SAT	2.710, −0.366, 4.56	7.44, 10.52, 3.09	—

Note: $n_i = 1$ is used in all cases.
* $\lambda_i = 5, 5$.
**$\lambda_i = 15, 15, 3$.

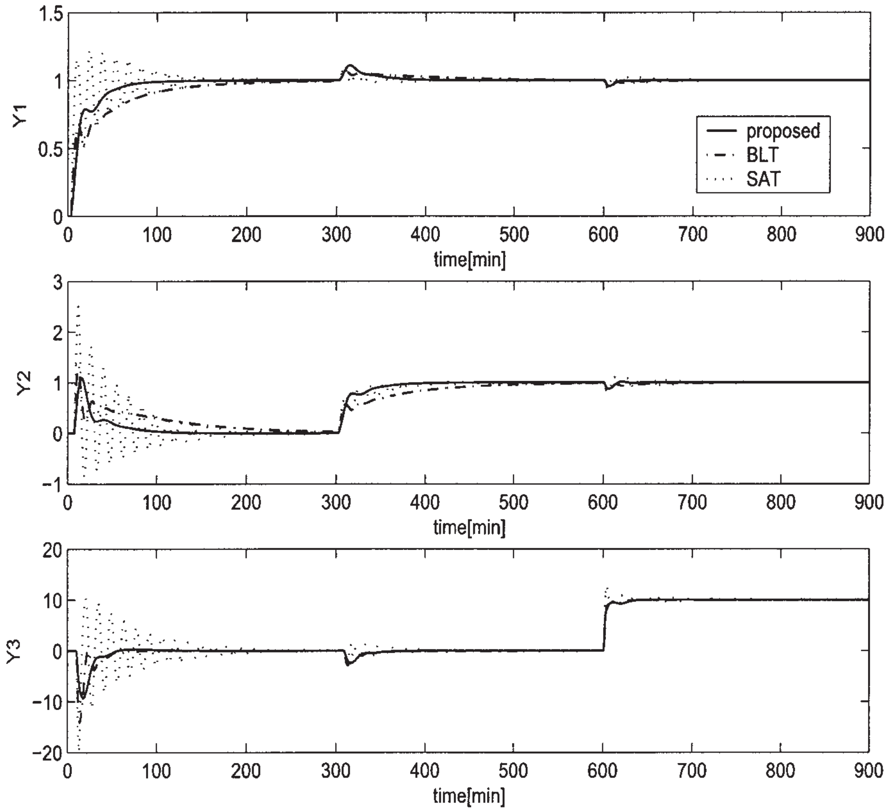

Figure 4. Closed-loop responses to sequential step changes in set point for Ogunnaike and Ray column.

$$K_i(s) = \frac{1}{s}\left[f_i(0) + f_i'(0)s + \frac{f_i''(0)}{2}s^2 + O(s^3)\right] \tag{11}$$

where $f_i(s) = K_i(s)s$.

By truncating the higher-order s terms, except the first three terms, the resulting ideal controller can be interpreted as the standard PID controller. Finally K_{ci} and τ_{Di} of the multiloop PID controller can be obtained by

$$K_{ci} = f_i'(0); \quad \tau_{Di} = f_i''(0)/2K_{ci} \tag{12}$$

Design of integral time constant τ_I

Because the integral term $\tilde{\mathbf{K}}_0$ in the PID control algorithm is dominating at low frequencies, it needs to be designed based on the interaction characteristics at low frequencies. It is clear from Eq. 1 that the loop interactions cannot be neglected at low frequencies. Therefore, $\tilde{\mathbf{K}}_0$ should be designed by taking the off-diagonal terms of $\mathbf{G}(s)$ into account. Comparing each diagonal element of the first-order s terms in Eqs. 6 and 7 gives an analytical tuning rule for τ_{Ii} as

$$\tau_{Ii} = -\frac{[G_{ii+}'(0) - n_i\lambda_i]K_{ci}}{[\mathbf{G}^{-1}(0)]_{ii}} \tag{13}$$

Simulation Study

Example 1

The distillation column of Wood and Berry (1973) was studied.

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{12.8\exp(-s)}{16.7s + 1} & \dfrac{-18.9\exp(-3s)}{21s + 1} \\ \dfrac{6.6\exp(-7s)}{10.9s + 1} & \dfrac{-19.4\exp(-3s)}{14.4s + 1} \end{bmatrix}$$

To evaluate how closely the proposed multiloop controller approximates the ideal multiloop controller, the Bode diagrams were drawn for the proposed PI controller and the ideal controller. The result is given in Figure 2. As can be seen in the figure, the proposed PI controller is closely approximated to the ideal controller over the whole frequency range, which illustrates the validity of our approach to use the frequency-dependent interaction characteristics. This close approximation essentially leads to satisfactory control performance. Figure 3 shows the closed-loop responses by several tuning methods. Control performances of the proposed PI and PID controllers were compared with those by the BLT method (Luyben, 1986) and the SAT method (Loh et al., 1993). To evaluate both set-point tracking and disturbance rejection performances, unit step changes in set point were sequentially made in the individual loops. n_i was chosen as 1 for all the loops according to the process model order. All the controller parameters used in the example are listed in Table 1. The proposed controllers show satisfactory performances. Note that both the proposed PI and PID controllers give almost identical performances because, in this example, the close approximation is already established with the PI action only.

Example 2

The 3 × 3 MIMO system (Ougnnaike et al., 1983) was considered.

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{0.66\exp(-2.6s)}{6.7s+1} & \dfrac{-0.61\exp(-3.5s)}{8.64s+1} & \dfrac{-0.0049\exp(-s)}{9.06s+1} \\ \dfrac{1.11\exp(-6.5s)}{3.25s+1} & \dfrac{-2.36\exp(-3s)}{5s+1} & \dfrac{-0.012\exp(-1.2s)}{7.09s+1} \\ \dfrac{-33.68\exp(-9.2s)}{8.15s+1} & \dfrac{46.2\exp(-9.4s)}{10.9s+1} & \dfrac{0.87(11.61s+1)\exp(-s)}{(3.89s+1)(18.8s+1)} \end{bmatrix}$$

Figure 4 compares the closed-loop response of the proposed multiloop PI controller with those by the BLT method and the SAT method. Sequential step changes of magnitude 1, 1, and 10 in set point were made to the 1st, 2nd, and 3rd loops, respectively. The controller parameters used in the simulation are also listed in Table 1. The superior performance of the proposed method is readily apparent.

Conclusions

An analytical method for multiloop PID controller design is proposed by extending the generalized IMC-PID tuning method for SISO systems. Simple but efficient tuning rules can be obtained by using the frequency-dependent properties of the closed-loop interactions: the proportional and derivative terms are designed simply by neglecting the off-diagonal elements, whereas the integral term is designed by taking the off-diagonal elements fully into account. The resulting multiloop PID controller closely approximates the ideal multiloop controller over the whole frequency range and, in turn, gives satisfactory control performance.

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