Consider the generalized IMC-PID method for PID controller tuning of time-delay processes

This simple analytical method provides PID parameters to give a desired closed-loop response while available for any class of time-delay processes

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Because the PID controller finds widespread use in the process industries, a great deal of effort has been directed at finding the best choices for the controller gain, integral and derivative time constants for various process models. Among the various PID tuning methods, IMC-PID1,2 has gained widespread acceptance in the chemical process industries because of its simplicity, robustness and successful practical applications.

In most time-delay process cases, the ideal controller that gives the desired closed-loop response is more complicated than a PID controller. In the IMC-PID tuning methods, this problem is solved by using clever approximations of the time-delay term in such a way that the controller form can be reduced to that of a PID controller, or a PID controller cascaded with a first- or second-order lag. The approach often causes performance degradation of resulting PID controllers due to approximation inaccuracies and introduces an unnecessary additional lag filter. Furthermore, the tuning rule is available only for a restricted class of process models that yield the PID structures by the approximations.

Lee, et al.,3 suggested the generalized IMC-PID tuning method to cope with any class of time-delay process models under the unified framework. In the proposed method, the PID parameters are obtained by approximating the ideal controller with a Maclaurin series in the Laplace variable. Therefore, the generalized IMC-PID method proposed has no restriction on the class of process models. In addition, it turns out that the PID parameters so obtained provide somewhat better closed-loop responses than those obtained previously. The analytical form of the resulting tuning rules is also practically very attractive.

In this article, tuning rules based on the generalized IMC-PID tuning method are presented for various processes such as stable, unstable and integrating processes. Tuning rules for cascade systems are also presented.

Generalized IMC-PID method. A classical feedback diagram is shown in Fig. 1. The process response to inputs is:

\[ C = \frac{G_c G}{1 + G_c G} q_s R + \frac{G_p}{1 + G_c G} d \]

where \( R \) denotes the setpoint and \( q_s \) denotes the setpoint filter.

In Eq. 1, the process model can be generally represented as:

\[ G(s) = p_m(s) p_A(s) \]

where \( p_m(s) \) is the portion of the model inverted by the controller, and \( p_A(s) \) is the portion of the model not inverted by the controller (time delay, inverse process) and \( p_A(0) = 1 \).

Our aim is to design the controller \( G_c \) of Fig. 1 in such a way as to give the desired closed-loop response of:

\[ \frac{C}{R} = \frac{p_A(s)}{(\lambda s + 1)^r} \]

The term \( 1/(\lambda s + 1)^r \) functions as a filter with an adjustable time constant, \( \lambda \), and an order, \( r \), is chosen so that the controller, \( G_c \), is realizable. Note that \( \lambda \) is analogous to the closed-loop time constant.

The controller \( G_c \) that gives the desired loop response given by Eq. 3 perfectly is then written by: Continued
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The controller given by Eq. 5 can be approximated to the PID controller by using only the first three terms: \(1/s, 1 \) and \( s \) in Eq. 6 and truncating all other high-order terms \( (s^2, s^3, \ldots) \). The first three terms of the expansion can be interpreted as the standard PID controller given by:

\[
G_i(s) = K_C \left( 1 + \frac{1}{\tau_i s} + \tau_D s \right)
\]

where

\[
K_C = \frac{f''(0)}{f'(0)}
\]

\[
\tau_i = \frac{f''(0)}{f'(0)}
\]

\[
\tau_D = \frac{f''(0)}{2f'(0)}
\]

The controller can also be obtained from Eq. 8 in a straightforward manner. The integral and/or derivative time constants, \( \tau_i, \tau_D \), from Eq. 8 usually have positive values. A few processes have strong lead terms and thus show significant overshoots in response to step changes in the input. In this case, it might be extremely difficult for the process to give a desired overdamped response with a simple PID controller alone. Therefore, the PID controller cascaded with a low-pass filter such as \( \frac{1}{\alpha^2 s^2 + \alpha s + 1} \) or \( \frac{1}{C(s+1)} \) is recommended to compensate for the effect of the lead term. Tuning rules for the PID parameters and the filter time constants for this case are also available based on the proposed approach (see Lee, et al., for more details).

**Tuning rules for FOPDT and SOPDT models.** The most commonly used approximate models for chemical processes are the first-order plus dead-time (FOPDT) model and/or the second-order plus dead-time (SOPDT) model given as:

**FOPDT:**

\[
G(s) = \frac{K e^{-\theta t}}{\tau s + 1}
\]

**SOPDT:**

\[
G(s) = \frac{K e^{-\theta t}}{(\tau_1 s + 1)(\tau_2 s + 1)}
\]

Tuning rules for the two typical models are shown in Table 1 where \( q_i = 1 \). Note that the tuning rule for the SOPDT model is available not only for the overdamped systems but also for the underdamped systems.

In this method, the closed-loop time constant, \( \lambda \), is used as a tuning parameter to adjust the speed and robustness of the closed-loop system. Extensive simulation has been done to find the best value of \( \lambda/\theta \) in the sense of robustness and performance. As a result, \( \lambda/\theta = 0.5 \) is recommended as a practical guideline for a good starting value. For small \( \theta/\tau \) (typically less than 0.2), a detuning might be considered to account for constraints on manipulated variables. As the model uncertainty increases, \( \lambda \) should increase accordingly. Note that the closed-loop response becomes sluggish as \( \lambda \) increases.

**Example.** As an example, consider a process with the SOPDT model as:

\[
G_i(s) = \frac{1}{s} \left[ f(0) + f''(0)s + \frac{f'''(0)}{2} s^2 + \ldots \right]
\]
Fig. 2 compares the closed-loop responses by the generalized IMC-PID and Smith\textsuperscript{4} methods. The resulting PID controller by the proposed method performs better than the controller tuned by the Smith method.

Tuning rules for other complicated time-delay models. One of the main advantages of the proposed method is that it has no restriction on the class of process models. Tuning rules by the generalized IMC-PID method for the several complicated process models such as integrating processes, distributed parameter processes and inverse processes with time delays are also listed in Table 2.

Tuning rules for unstable systems. Many unstable processes still exist in chemical plants, even though most chemical processes are open-loop stable. The most common example is the batch chemical reactor, which has a strong instability due to the heat generation term in the energy balance. Two representative types of time-delayed unstable processes are the first-order delayed unstable process (FODUP) and the second-order delayed unstable process (SODUP).

### TABLE 1. Generalized IMC-PID tuning rules for FOPDT and SOPDT processes

<table>
<thead>
<tr>
<th>Process model</th>
<th>( K_C )</th>
<th>( \tau_i )</th>
<th>( \tau_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOPDT</td>
<td>( \frac{K e^{-\tau s}}{\tau s + 1} )</td>
<td>( \frac{\tau_i}{K(\lambda + \theta)} )</td>
<td>( \frac{\theta_i^2}{2(\lambda + \theta)} )</td>
</tr>
<tr>
<td>SOPDT</td>
<td>( \frac{K e^{-\tau s}}{(\tau^2 s^2 + 2\tau s + 1)} )</td>
<td>( \frac{\tau_i}{K(\lambda + \theta)} )</td>
<td>( \frac{\theta_i^2}{2(\lambda + \theta)} )</td>
</tr>
</tbody>
</table>

Note: Desired closed-loop response \( \frac{C}{R} = \frac{e^{-\tau s}}{\tau s + 1} \), \( r = 1 \) and 2 for the FOPDT and SOPDT model, respectively.

### TABLE 2. Generalized IMC-PID tuning rules for various complicated processes

<table>
<thead>
<tr>
<th>Process model</th>
<th>( K_C )</th>
<th>( \tau_i )</th>
<th>( \tau_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrating process 1</td>
<td>( \frac{K}{s} )</td>
<td>( \frac{1}{K(\lambda + \theta)} )</td>
<td>( \frac{\theta_i^2}{2(\lambda + \theta)} )</td>
</tr>
<tr>
<td>Integrating process 2</td>
<td>( \frac{K}{s(\tau s + 1)} )</td>
<td>( \frac{1}{K(\lambda + \theta)} )</td>
<td>( \tau_i + \frac{\theta_i^2}{2(\lambda + \theta)} )</td>
</tr>
<tr>
<td>Distributed parameter process</td>
<td>( \frac{K(\tau s + 1)e^{-\tau s}}{(\tau^2 s^2 + 2\tau s + 1)} )</td>
<td>( \frac{\tau_i}{K(\lambda + \theta)} )</td>
<td>( \frac{\theta_i^2}{2(\lambda + \theta)} )</td>
</tr>
<tr>
<td>Inverse process 1</td>
<td>( \frac{K(-\tau s + 1)e^{-\tau s}}{(\tau s + 1)} )</td>
<td>( \frac{\tau_i}{K(\lambda + \theta + 2\tau_s)} )</td>
<td>( \frac{\theta_i^2}{2(\lambda + \theta + 2\tau_s)} )</td>
</tr>
<tr>
<td>Inverse process 2</td>
<td>( \frac{K(-\tau s + 1)e^{-\tau s}}{s(\tau s + 1)} )</td>
<td>( \frac{1}{K(\lambda + \theta + 2\tau_s)} )</td>
<td>( \tau_i + \frac{\theta_i^2}{2(\lambda + \theta + 2\tau_s)} )</td>
</tr>
<tr>
<td>Inverse process 3</td>
<td>( \frac{K(-\tau s + 1)e^{-\tau s}}{(\tau^2 s^2 + 2\tau s + 1)} )</td>
<td>( \frac{\tau_i}{K(\lambda + \theta + 2\tau_s)} )</td>
<td>( \frac{\theta_i^2}{2(\lambda + \theta + 2\tau_s)} )</td>
</tr>
</tbody>
</table>

Note: Desired closed-loop response \( \frac{C}{R} = \frac{e^{-\tau s}}{\tau s + 1} \) for the inverse processes.
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FODUP: \( G(s) = \frac{Ke^{-\theta s}}{\tau s - 1} \)

SODUP: \( G(s) = \frac{Ke^{-\theta s}}{(\tau s - 1)(as + 1)} \)

The generalized IMC-PID approach can be extended to integrating and unstable processes. Additionally, a setpoint filter, \( q_s \), shown in Fig. 1 is designed not to give overshoots in servo problems. Most unstable processes in the process industries can be modeled unstable processes with one RHP pole (FODUP and SODUP), unstable processes with two RHP poles and integrating unstable processes. Tuning rules based on the generalized IMC-PID method for these processes are listed in Table 3. In the case where the offset by the tuning rules in Table 2 is critical for integrating processes, consider the tuning rules in Table 3 because we can design the PID controllers by considering the integrating processes as the FODUP or SODUP model (see Lee, et al., for more details). An extensive simulation indicates \( \lambda/\theta = 1 - 2 \) as a practical guideline for \( \lambda \).

**Example.** As an example, the following process is considered:

\[ G(s) = G_D(s) = \frac{e^{-0.4s}}{s - 1} \]  

(10)

Figs. 3a and 3b show the closed-loop responses of the unstable process given by Eq. 10 to a unit step change in setpoint, \( R \), and load, \( d \). The results shown in the figures illustrate the superior performance of the generalized IMC-PID method.

**Tuning rules for cascade systems.** Cascade control as shown in Fig. 4 is one of the most successful methods for enhancing single-loop control performance, particularly when the disturbances are associated with the manipulated variable or when the final control element exhibits nonlinear behavior. This important benefit has led to the extensive use of cascade

**TABLE 3. Generalized IMC-PID tuning rules for FODUP and SODUP processes**

<table>
<thead>
<tr>
<th>Process model</th>
<th>Process model</th>
<th>( K_C )</th>
<th>( \tau_I )</th>
<th>( \tau_D )</th>
<th>Setpoint filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>FODUP</td>
<td>( Ke^{-\theta s} / (\tau s - 1) )</td>
<td>( \tau_s )</td>
<td>( -\tau + \alpha + \alpha )</td>
<td>( -\tau + \alpha + \alpha )</td>
<td>( -\alpha )</td>
</tr>
<tr>
<td>SODUP (a)</td>
<td>( (\tau s - 1)(as + 1) )</td>
<td>( \tau_s )</td>
<td>( -\tau + \alpha + \alpha )</td>
<td>( -\tau + \alpha + \alpha )</td>
<td>( -\alpha )</td>
</tr>
<tr>
<td>SODUP (b)</td>
<td>( (\tau s - 1)(\tau s - 1) )</td>
<td>( \tau_s )</td>
<td>( -\tau + \alpha + \alpha )</td>
<td>( -\tau + \alpha + \alpha )</td>
<td>( -\alpha )</td>
</tr>
</tbody>
</table>

where \( \alpha = [\tau / (\tau + 1)e^{\tau s} - 1] \), desired closed-loop response is \( C/R = e^{-\alpha s} / (\lambda s + 1) \) in FODUP and SODUP(a); \( \alpha, \tau \) values are calculated by solving \( 1 - \frac{(\alpha, \tau; \alpha, \tau + 1)e^{-\alpha s}}{(\lambda s + 1)^{\tau}} \) = 0.

desired closed-loop response is \( C/R = e^{-\alpha s} / (\lambda s + 1) \) in SODUP(b).
TABLE 4. Generalized IMC-PID tuning rules for cascade control systems

<table>
<thead>
<tr>
<th>Process model</th>
<th>Reference trajectory</th>
<th>$K_C$</th>
<th>$\tau_1$</th>
<th>$\tau_2$</th>
<th>$T_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOPDT (inner-loop)</td>
<td>$\frac{K_r e^{-\theta y}}{\tau_s s + 1}$</td>
<td>$e^{-\frac{\theta y}{\lambda_s s + 1}}$</td>
<td>$\frac{\tau_1}{K_C(\lambda_s + \theta_s)}$</td>
<td>$\tau_1 + \frac{\theta_2^2}{2(\lambda_s + \theta_s)}$</td>
<td>$\frac{6(\lambda_s + \theta_s)}{\theta_2^4 (3 - \frac{\theta_2^2}{\tau_1})}$</td>
</tr>
<tr>
<td>SOPDT (inner-loop)</td>
<td>$\frac{K_r e^{-\theta y}}{(\tau_s^2 s^2 + 2\theta_2 \tau_s s + 1)}$</td>
<td>$e^{-\frac{\theta y}{\lambda_s s + 1}}$</td>
<td>$\frac{\tau_1}{K_C(\lambda_s + \theta_s)}$</td>
<td>$2\tau_2 \tau_1 + \frac{\theta_2^2}{2(\lambda_s + \theta_s)}$</td>
<td>$\frac{6(\lambda_s + \theta_s)}{\theta_2^4 (3 - \frac{\theta_2^2}{\tau_1})}$</td>
</tr>
<tr>
<td>FOPDT (outer-loop)</td>
<td>$\frac{K_r e^{-\theta y}}{\tau_s s + 1}$</td>
<td>$\frac{\tau_1}{K_C(\lambda_s + \theta_s)}$</td>
<td>$\tau_1 + \frac{\theta_2^2}{2(\lambda_s + \theta_s)}$</td>
<td>$\frac{(\lambda_s + \theta_s - \frac{\theta_2^2}{\tau_1})}{(\lambda_s + \theta_s + \frac{\theta_2^2}{\tau_1})}$</td>
<td>$\frac{(\lambda_s + \theta_s + \frac{\theta_2^2}{\tau_1})}{(\lambda_s + \theta_s - \frac{\theta_2^2}{\tau_1})}$</td>
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<td>SOPDT (outer-loop)</td>
<td>$\frac{K_r e^{-\theta y}}{(\tau_s^2 s^2 + 2\theta_2 \tau_s s + 1)}$</td>
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</tr>
</tbody>
</table>

control in the chemical process industries. The generalized IMC-PID method was extended to cascade control systems. Tuning rules based on the generalized IMC-PID method for FOPDT and SOPDT in cascade control systems are shown in Table 4. $\lambda_1/(\theta_1 + \theta_2) = 0.5$ and $\lambda_2/\theta_2 = 0.5$ are recommended as a practical guideline for $\lambda$.

**Example.** As an example to evaluate the robustness against a structural mismatch in the plant model, the following complicated process was tested:

$$G_{p1} = \frac{10(-5s + 1)e^{-5s}}{(30s + 1)^3(10s + 1)}$$

$$G_{p2} = \frac{3e^{-3s}}{13s + 1}$$

$$G_{p1} = \frac{e^{-10s}}{100s^2 + 20s + 1}, G_{p2} = \frac{1}{100s + 1}$$

We added white noises to $C_2$ and $C_1$ to reflect the noise effect from real process measurements. We identified the processes both in the inner and the outer loops with the FOPDT model. The reduced models were obtained by minimizing squared error between the process output data and the model output data. We obtained the reduced process models as:

$$G_{r1} = \frac{102e^{-6.71s}}{66.49s + 1}, G_{r2} = \frac{2.988e^{-3.66s}}{13.28s + 1}$$

The PID controllers were tuned by the proposed method with $\lambda_1 = 30.85$ and $\lambda_2 = 1.83$.

Fig. 5 shows the closed-loop responses tuned by the generalized IMC-PID method and the ITAE method for load changes in $L_2$. The superior performance of the generalized IMC-PID method is readily apparent.

**LITERATURE CITED**
