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Analytical method of PID controller design for parallel cascade control

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Abstract

An analytical method of PID controller design is proposed for parallel cascade control. Firstly, a general structure for parallel cascade control is proposed that takes both setpoint and load disturbance responses into account. Analytical tuning rules for the PID controllers are then derived for the general process model by employing the IMC design procedure. The proposed method offers a simple and effective way to obtain the PID controller rules for parallel cascade control system which takes into account the interaction between primary and secondary control loops. The simulation results illustrate the application of the proposed method and demonstrate its superiority compared to several alternatives.

Keywords: Parallel cascade control; PID controller tuning; IMC controller design; Two-degree-of-freedom controller; Analytical tuning rules

1. Introduction

In process industries, cascade control is widely used to reduce the effects of possible disturbances and to improve the dynamic performance of the closed-loop system. In the traditional series cascade control, both the manipulated variable and the disturbance affect the primary output through the intermediate (secondary) output. The cascade control of the parallel type (Fig. 1) arises when the manipulated and disturbance variables simultaneously affect primary and secondary outputs (y_1 and y_2). Parallel cascade control was first considered by Luyben [1]. The overhead composition control of a distillation column (Fig. 2), cascaded onto the control of a tray temperature, is a typical example of a parallel cascade control system. The reflux flow rate (manipulated variable) and the feed flow or composition (disturbance, d) affect, both, the purity of the overhead product (primary output, y_1) and the tray temperature (secondary output, y_2). The control objective is to maintain the overhead composition at the setpoint. The output of the composition controller resets the setpoint for the temperature controller. By controlling the tray temperature in the cascade manner, the variation in the feed can be compensated before it disturbs the product composition. In general, parallel cascade control is appropriate when the secondary loop has a faster dynamic response and the rejection of the disturbance in the secondary output reduces the steady state output error in the primary loop. The parallel cascade control is also beneficial when measurements of the primary output are sampled infrequently and/or with long time delays.

Despite clear benefits of the parallel cascade control and its wide-spread use in process industries, the design on the parallel cascade control systems has attracted relatively little research. Yu [2] proposed an efficient interaction measure to determine whether the parallel cascade control is advantageous or detrimental to load response. He also used the index to quantify the margin of improvement over conventional single loop control. Shen and Yu [3] applied these results to the selection of the secondary measurement in parallel cascade control. Brambilla and Semino [4] introduced a nonlinear filter in order to partially separate the

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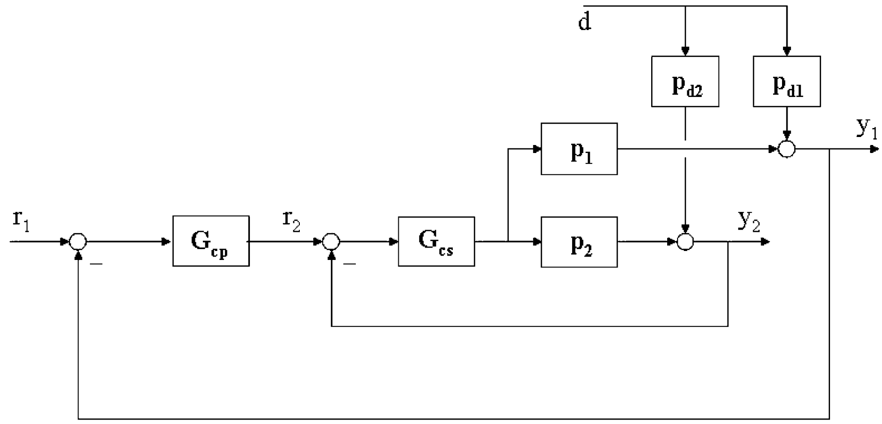

Fig. 1. Parallel cascade system.

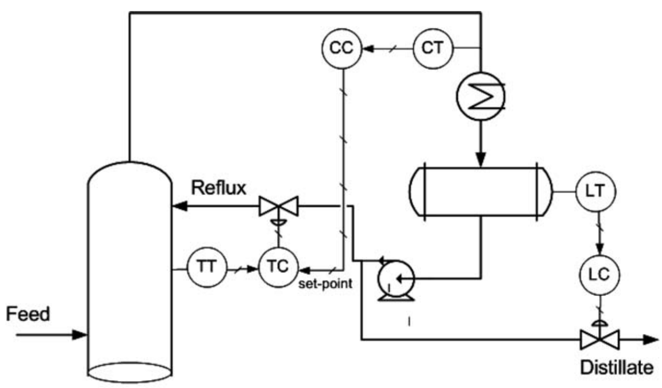

Fig. 2. Example of parallel cascade control: an overhead composition control in the distillation column.

dynamics of the primary and secondary loop in parallel cascade control. They also developed a combined structure to avoid interactions between the primary and secondary loops. Specifically, they proposed to use a conventional feedback controller in the secondary loop and an IMC controller in the primary loop [5].

A design issue that has not yet been addressed in prior research is the tradeoff (water-bed effect) between the performance of the setpoint and load responses in parallel cascade control systems. For example, disturbance rejection in the primary loop depends on the disturbance rejection and setpoint tracking in the secondary loop. In the conventional parallel cascade system, when the disturbance rejection in the secondary loop is optimized, the setpoint response is often found to be poor (and vice versa), which may lead to the deterioration of the disturbance rejection in the primary loop. The difficulty of simultaneously achieving good disturbance rejection and setpoint tracking also occurs in the design of the primary controller.

Another issue is the tuning of PID controllers. For a given parallel cascade control structure, closed-loop performance is determined by tuning the PID controllers in both loops. The simplest solution of independently tuning primary and secondary controllers based on the corresponding process models, p_1 and p_2, is often ineffective because it ignores strong interaction between the two loops. A widely used alternative is to use a two-step approach similar to the standard procedure for tuning of the series cascade system. First, the secondary controller is tuned based on the dynamic model of the secondary process with the primary controller in manual mode. The primary controller is then tuned using the dynamic model obtained with the secondary loop in automatic. In this approach, however, if the secondary controller is retuned for some reason, an additional identification step is essential for retuning the primary controller, which is often cumbersome in practice [6].

In this paper, we propose a general approach to cope with both setpoint tracking and disturbance rejection in the parallel cascade control. The IMC design is used to obtain controllers for each loop. The designed IMC controllers are then approximated by the PID controllers. The tuning rules for primary and secondary PID controllers are summarized in the analytical form, which simplifies the implementation. Numerical simulations are used to compare different designs.

2. General structure for parallel cascade control and corresponding IMC structure

The parallel cascade control structure is shown in Fig. 1, where p and G refer to the transfer functions of the process and the controller, respectively; r, d and y are setpoint, disturbance and output variables. The disturbance rejection in the primary loop depends on, both, disturbance rejection and setpoint tracking in the secondary control loop. When disturbance rejection (setpoint tracking) in the secondary loop is optimized, the corresponding setpoint response (disturbance rejection) may be poor, which leads to the degradation of disturbance rejection in the primary loop. To address this difficulty, we propose to use two-degree-of-freedom (2DOF) controllers in both loops, where q_{f1} and q_{f2} in Fig. 3 are setpoint filters, and develop an IMC-based procedure for design of primary and secondary controllers.

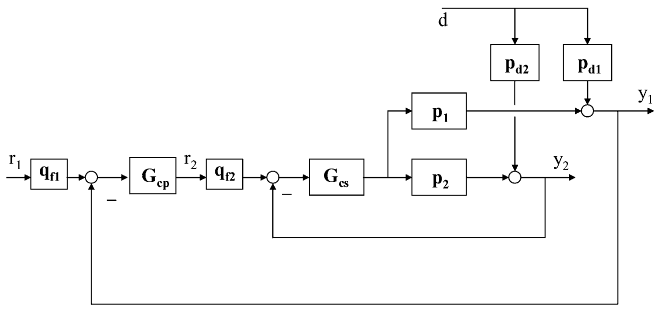

Fig. 3. General structure for a parallel cascade control system.

The IMC implementation of the general parallel cascade control structure is shown in Fig. 4, where q refers to the transfer function of the IMC controller and $\tilde{p}$ refers to the transfer function of the process model.

For the nominal case of no plant-model mismatch, the secondary and the primary outputs for the system in Fig. 4 are equal to

$$y_2 = p_2 q_2 q_{f2} r_2 + (1 - p_2 q_2) p_{d2} d \tag{1}$$

$$y_1 = q_2 q_{f2} p_1 q_1 q_{f1} r_1 + (1 - q_2 q_{f2} p_1 q_1)(p_{d1} - p_{d2} q_2 p_1) d \tag{2}$$

Note that the primary process model p_p is equal to $q_2 q_{f2} p_1$. We assume the nominal case throughout this paper.

3. IMC controller design

3.1. IMC controller for the secondary loop

According to the IMC parameterization [7], the process model of the secondary loop is decomposed into two parts:

$$p_2(s) = p_{2m}(s) p_{2a}(s) \tag{3}$$

where p_{2m} is the part of the model inverted by the controller and p_{2a} is the portion of the model that controller does not attempt to invert. The noninvertible part usually includes the portion of model with time delays and right half plane zeros and has an allpass form.

3.1.1. Case A: process contains poles sufficiently slower than the desired closed-loop response

To achieve fast closed-loop response for a process with slow open-loop dynamics, the IMC controller q_2 should satisfy the following conditions:

- If the process model p_2 has slow poles at $z_1, z_2, \ldots, z_m$, then q_2 should have zeros at $z_1, z_2, \ldots, z_m$.
- If the process model p_{d2} has slow poles at $z_{d1}, z_{d2}, \ldots, z_{dm}$, then $1 - p_2 q_2$ should have zeros at $z_{d1}, z_{d2}, \ldots, z_{dm}$.

Since the IMC controller q_2 is designed as $q_2 = p_{2m}^{-1} f_2$, the first condition is satisfied automatically. The second condition can be satisfied by requiring that the IMC filter f_2 satisfies the following equality:

$$f_2 = \frac{\sum_{i=1}^{m} \alpha_i s^i + 1}{(\lambda_2 s + 1)^{r2}} \tag{4}$$

where λ_2 is the adjustable parameter which controls the tradeoff between performance and robustness of the secondary loop; r_2 is selected large enough to make the IMC controller q_2 (semi-)proper; m is the number of dominant poles to be canceled and usually on the order of one or two; α_i's are determined from Eq. (5) to cancel the dominant poles in p_{d2}:

$$1 - p_2 q_2|_{s=z_{d1},\ldots z_{dm}} = \left| 1 - \frac{p_{2a}(\sum_{i=1}^{m} \alpha_i s^i + 1)}{(\lambda_2 s + 1)^{r2}} \right|_{s=z_{d1},\ldots,z_{dm}} = 0 \tag{5}$$

The designed internal model controller

$$q_2 = p_{2m}^{-1} f_2 = p_{2m}^{-1} \frac{\sum_{i=1}^{m} \alpha_i s^i + 1}{(\lambda_2 s + 1)^{r2}} \tag{6}$$

results in the following closed-loop response of the secondary output:

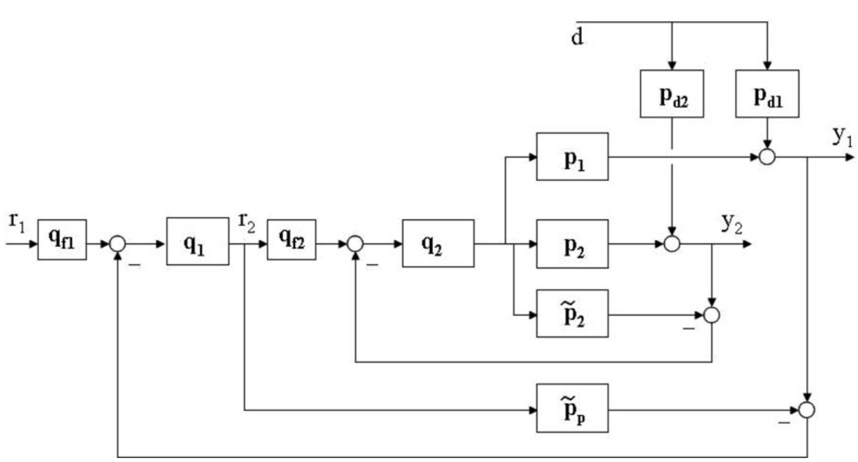

Fig. 4. IMC implementation of the general structure for a parallel cascade control system.

$$y_2 = \frac{p_{2a}(\sum_{i=1}^{m}\alpha_i s^i + 1)}{(\lambda_2 s+1)^{r2}} q_{f2} r_2 + \left(1 - \frac{p_{2a}(\sum_{i=1}^{m}\alpha_i s^i + 1)}{(\lambda_2 s+1)^{r2}}\right) p_{d2} d \tag{7}$$

The lead term in transfer function for the closed-loop setpoint response may cause an excessive overshoot and large settling time, which may lead to the degradation of the primary loop performance. To eliminate the overshoot, the setpoint filter q_{f2} is designed as

$$q_{f2} = \frac{1}{\sum_{i=1}^{m}\alpha_i s^i + 1} \tag{8}$$

which gives the following expression for the output of the secondary loop:

$$y_2 = \frac{p_{2a}}{(\lambda_2 s+1)^{r2}} r_2 + \left(1 - \frac{p_{2a}(\sum_{i=1}^{m}\alpha_i s^i + 1)}{(\lambda_2 s+1)^{r2}}\right) p_{d2} d \tag{9}$$

It is apparent that λ_2 is the desired closed-loop time constant for the setpoint response of the secondary loop. A smaller λ_2 provides faster response but at the expense of a more active control inputs; with a larger λ_2 the control is less aggressive but the closed-loop response is slower.

3.1.2. Case B: all open-loop poles are faster than the desired closed-loop response

For the open-loop process dynamics that is as fast as the desired closed-loop response, there is little benefit in pole cancellation introduced by the IMC design. In fact, such cancellation unjustifiably complicates the controller, which may cause larger errors when it is later approximated by a PID controller. Therefore, in case B, the IMC and the setpoint filters are designed simply as

$$f_2 = \frac{1}{(\lambda_2 s+1)^{r2}}; \quad q_{f2} = 1 \tag{10}$$

This is the case of a conventional 1DOF parallel cascade structure, for which the secondary output is obtained by setting $\alpha_i = 0$ in Eq. (9).

3.2. IMC controller for the primary loop

With the designed secondary controller, a primary process model p_p used in the design of the primary IMC controller becomes

$$p_p = q_2 q_{f2} p_1 = \frac{p_{2m}^{-1}}{(\lambda_2 s+1)^{r2}} p_1 \tag{11}$$

which is then factored as

$$p_p = p_{pm} p_{pa} \tag{12}$$

where the invertible part is

$$p_{pm} = \frac{p_{2m}^{-1} p_{1m}}{(\lambda_2 s+1)^{r2}} \tag{13}$$

while the noninvertible part p_{pa} is equal to p_{1a} which is the noninvertible part of p_1.

The primary output, Eq. (2), can now be expressed as

$$y_1 = \frac{p_{2m}^{-1}}{(\lambda_2 s+1)^{r2}} p_1 q_1 q_{f1} r_1 + \left(1 - \frac{p_{2m}^{-1}}{(\lambda_2 s+1)^{r2}} p_1 q_1\right) p_{dp} d \tag{14}$$

where $p_{dp} = p_{d1} - p_{d2} q_2 p_1$.

3.2.1. Case A: process contains poles sufficiently slower than the desired closed-loop response

This is the case when the dominant open-loop dynamics of the primary process is significantly slower than the desired closed-loop dynamics. With the same logic used in the secondary loop in Section 3.1, the filter f_1 should be designed as

$$f_1 = \frac{\sum_{i=1}^{n}\beta_i s^i + 1}{(\lambda_1 s+1)^{r1}} \tag{15}$$

where λ_1 is the filter parameter for adjusting performance and robustness of the primary loop; r_1 is selected large enough to make the primary IMC controller (semi)proper; n is the number of poles to be canceled, which in many practical situations is equal one or two; β_i is determined to cancel the dominant poles $v_{d1}, v_{d2}, \ldots, v_{dn}$ in p_{dp} and is found from the solution of the following equation:

$$1 - p_p q_1|_{s=v_{d1},\ldots v_{dn}} = \left|1 - \frac{p_{pa}(\sum_{i=1}^{n}\beta_i s^i + 1)}{(\lambda_1 s+1)^{r1}}\right|_{s=v_{d1},\ldots v_{dn}} = 0 \tag{16}$$

The primary IMC controller is then equal to

$$q_1 = p_{pm}^{-1} \frac{(\sum_{i=1}^{n}\beta_i s^i + 1)}{(\lambda_1 s+1)^{r1}} \tag{17}$$

and the closed-loop transfer function describing the response of the primary loop to the setpoint change becomes

$$\frac{y_1}{r_1} = p_p q_1 q_{f1} = \frac{p_{pa}(\sum_{i=1}^{n}\beta_i s^i + 1)}{(\lambda_1 s+1)^{r1}} q_{f1} \tag{18}$$

An overshoot caused by the lead term in Eq. (18) can be eliminated by designing the setpoint filter, q_{f1}, as

$$q_{f1} = \frac{1}{\sum_{i=1}^{n}\beta_i s^i + 1} \tag{19}$$

The primary output of the closed-loop system is then given by

$$y_1 = \frac{p_{pa}}{(\lambda_1 s+1)^{r1}} r_1 + \left(1 - \frac{p_{pa}(\sum_{i=1}^{n}\beta_i s^i + 1)}{(\lambda_1 s+1)^{r1}}\right) p_{dp} d \tag{20}$$

3.2.2. Case B: all open-loop poles are faster than the desired closed-loop response

Slow sampling rate and time delays in the measurements of primary output may limit practically achievable speed of the closed-loop response to that determined by the dominant process time constant. In this case, the IMC and the setpoint filters are selected as

$$f_1 = \frac{1}{(\lambda_1 s + 1)^{r1}}; \quad q_{f1} = 1 \tag{21}$$

The expression for the primary process output is obtained by setting $\beta_i = 0$ in Eq. (20).

4. PID controller design

In practical applications, the PID controllers are preferable to more complex designs. In this section, the developed IMC structure for the parallel cascade control system is approximated with PID controllers, and the analytical PID tuning rules are derived.

4.1. Equivalent feedback controller

The proposed IMC control system described in the previous section can be represented by the classical feedback structure as shown in Fig. 3. Here, the equivalent feedback controllers, G_c, are related to the IMC controllers as

$$G_c = \frac{q}{1 - pq} \tag{22}$$

Substitution of Eqs. (3) and (6) into (22) gives the equivalent feedback controller of the secondary loop:

$$G_{cs}(s) = \frac{p_{2m}^{-1}(\sum_{i=1}^{m} \alpha_i s^i + 1)}{(\lambda_2 s + 1)^{r2} - p_{2a}(\sum_{i=1}^{m} \alpha_i s^i + 1)} \tag{23}$$

Similarly, using Eqs. (12) and (17) in (22) obtains the equivalent feedback controller of the primary loop:

$$G_{cp}(s) = \frac{p_{pm}^{-1}(\sum_{i=1}^{n} \beta_i s^i + 1)}{(\lambda_1 s + 1)^{r1} - p_{pa}(\sum_{i=1}^{n} \beta_i s^i + 1)} \tag{24}$$

If either a secondary or primary loop belongs to case B, then the equivalent feedback controller is simplified by setting $\alpha_i = 0$ or $\beta_i = 0$.

4.2. PID approximation of the equivalent feedback controllers

The PID approximation of controllers (23) and (24) is obtained following the method of Lee et al. [8]. To simplify the notation, the subscripts s, p, indicating the secondary and primary loops, are dropped. Taking into the account that the steady state gain of noninvertible part of the process model is equal to 1, it is easy to verify by inspection that controllers given by in Eqs. (23) and (24) have a pole at the origin. Therefore, both controllers can be expressed as

$$G_c \equiv g(s)/s \tag{25}$$

The PID approximation of G_c can now be obtained using Maclaurin series approximation of g as a function of s:

$$G_c(s) = \frac{1}{s}\left(g(0) + g'(0)s + \frac{g''(0)}{2!}s^2 + \cdots + \frac{g^{(i)}(0)}{i!}s^i + \cdots\right) \tag{26}$$

The series approximation truncated to the first three terms can be interpreted as the standard PID controller given by

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s + \cdots\right) \tag{27}$$

where the analytical expressions for the PID constants are given by

$$K_c = g'(0); \ \tau_I = g'(0)/g(0); \quad \tau_D = g''(0)/2g'(0) \tag{28}$$

When process has a dominant lead term, the PID tuning according to Eq. (28) may lead to negative values of integral and derivative time constants irrespective of the selected filter time constant, λ. To address this problem, the equivalent feedback controller, G_c, can be approximated as a lagged PID controller: $K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)/(\tau_F s + 1)$.

To obtain the parameters of the PID · lag controller, we rewrite G_c as

$$G_c \equiv [g(s)(\tau_F s + 1)]/s(\tau_F s + 1) \tag{29}$$

By expanding $[g(s)(\tau_F s + 1)]$ into the Maclaurin series, one obtain:

$$G_c(s) = \frac{1}{s(\tau_F s + 1)}\{g(0) + [g'(0) + \tau_F g(0)]s + [g''(0) + 2\tau_F g'(0)]s^2/2! + [g'''(0) + 3\tau_F g''(0)]s^3/3! + \cdots\} \tag{30}$$

The third-order term in Eq. (30) is eliminated by setting the lag parameter, τ_F, equal to

$$\tau_F = -g'''(0)/3g''(0) \tag{31}$$

which gives the following tuning parameters for the lagged PID controller:

$$K_c = g'(0) + \tau_F g(0); \quad \tau_I = K_c/g(0); \quad \tau_D = [g''(0) + 2\tau_F g'(0)]/2K_c \tag{32}$$

An alternative way to approximate the equivalent feedback controller for the process with a dominant lead term using the PID · lag controller is to, first, obtain the PID controller from Eq. (28) for the process model with ignoring the lead term, and then add a lag with the time constant τ_F equal to the time constant of the ignored dominant lead term. The resulting PID · lag controller will be nearly the same as the one by Eqs. (31) and (32) while the more simple and compact tuning rule can be obtained.

4.3. PID controller tuning for FOPDT process

To illustrate the application of the developed method, consider a special case when both p_1 and p_2 are modeled as first-order plus dead time (FOPDT) processes:

$$p_2(s) = p_{d2}(s) = \frac{K_2 e^{-\theta_2 s}}{\tau_2 s + 1} \quad (33)$$

$$p_1(s) = p_{d1}(s) = \frac{K_1 e^{-\theta_1 s}}{\tau_1 s + 1} \quad (34)$$

Beginning with the secondary loop, factor the process model as

$$p_{2m} = \frac{K_2}{\tau_2 s + 1}; \quad p_{2a} = e^{-\theta_2 s} \quad (35)$$

For the case A, the IMC filter is

$$f_2 = \frac{\alpha s + 1}{(\lambda_2 s + 1)^2} \quad (36)$$

Therefore, the equivalent feedback controller of the secondary loop becomes

$$G_{cs} = \frac{(\tau_2 s + 1)(\alpha s + 1)}{K_2[(\lambda_2 s + 1)^2 - e^{-\theta_2 s}(\alpha s + 1)]} \quad (37)$$

For the case B, the IMC filter is simply set to

$$f_2 = \frac{1}{\lambda_2 s + 1} \quad (38)$$

and the equivalent feedback controller of the secondary loop is equal

$$G_{cs}(s) = \frac{\tau_2 s + 1}{K_2(\lambda_2 s + 1 - e^{-\theta_2 s})} \quad (39)$$

Expanding Eq. (37) (or Eq. (39)) into a Maclaurin series and using Eq. (28) gives the tuning rules for the secondary PID controller for the case A (or B). The result is summarized in Table 1.

Now, consider the primary PID controller. With the secondary controller designed, the primary model is given by Eq. (11), where

$$p_p = \frac{p_{2m}^{-1}}{(\lambda_2 s + 1)^{r2}} p_1 = \frac{\tau_2 s + 1}{K_2(\lambda_2 s + 1)^{r2}} \frac{K_1 e^{-\theta_1 s}}{\tau_1 s + 1} \quad (40)$$

Table 1
PID controller tuning rules for the FOPDT model in parallel cascade control

Secondary loop	Case A	Case B
Process model	$p_2 = \frac{K_2 e^{-\theta_2 s}}{\tau_2 s + 1}$	$p_2 = \frac{K_2 e^{-\theta_2 s}}{\tau_2 s + 1}$
Reference trajectory	$\frac{y_2}{r_2} = \frac{e^{-\theta_2 s}}{(\lambda_2 s + 1)^2}$	$\frac{y_2}{r_2} = \frac{e^{-\theta_2 s}}{\lambda_2 s + 1}$
K_c	$K_{cs} = \frac{\tau_{Is}}{K_2(2\lambda_2 + \theta_2 - \alpha)}$	$K_{cs} = \frac{\tau_{Is}}{K_2(\lambda_2 + \theta_2)}$
τ_I	$\tau_{Is} = \tau_2 + \alpha + \frac{\lambda_2^2 + \theta_2\alpha - \frac{1}{2}\theta_2^2}{2\lambda_2 + \theta_2 - \alpha}$	$\tau_{Is} = \tau_2 + \frac{\frac{1}{2}\theta_2^2}{\lambda_2 + \theta_2}$
τ_D	$\tau_{Ds} = \frac{\tau_2\alpha - \frac{\frac{1}{6}\theta_2^3 - \frac{1}{2}\theta_2^2\alpha}{2\lambda_2 + \theta_2 - \alpha}}{\tau_{Is}} - \frac{\lambda_2^2 + \theta_2\alpha - \frac{1}{2}\theta_2^2}{2\lambda_2 + \theta_2 - \alpha}$	$\tau_{Ds} = \frac{\theta_2^2}{2(\lambda_2 + \theta_2)}\left(1 - \frac{\frac{1}{3}\theta_2}{\tau_{Is}}\right)$
Set-point filter	$q_{f2} = \frac{1}{\alpha s + 1}; \quad \alpha = \tau_2\left(1 - \left(1 - \frac{\lambda_2}{\tau_2}\right)^2 e^{\frac{-\theta_2}{\tau_2}}\right)$	$q_{f2} = 1$
Primary loop	**Case A[a]**	**Case B[b]**
Process model	$p_1 = \frac{K_1 e^{-\theta_1 s}}{\tau_1 s + 1}$	$p_1 = \frac{K_2 e^{-\theta_1 s}}{\tau_1 s + 1}$
Reference trajectory	$\frac{y_1}{r_1} = \frac{e^{-\theta_1 s}}{(\lambda_1 s + 1)^2}$	$\frac{y_1}{r_1} = \frac{e^{-\theta_1 s}}{\lambda_1 s + 1}$
K_c	$K_{cp} = \frac{\tau_{Ip}}{\frac{K_1}{K_2}(2\lambda_1 + \theta_1 - \beta)}$	$K_{cp} = \frac{\tau_{Ip}}{\frac{K_1}{K_2}(\lambda_1 + \theta_1)}$
τ_I	$\tau_{Ip} = \tau_1 + \beta + \frac{\lambda_1^2 + \theta_1\beta - \frac{1}{2}\theta_1^2}{2\lambda_1 + \theta_1 - \beta}$	$\tau_{Ip} = \tau_1 + \lambda_2 - \tau_2 + \frac{\frac{1}{2}\theta_1^2}{\lambda_1 + \theta_1}$
τ_D	$\tau_{Dp} = \frac{\tau_1\beta - \frac{\frac{1}{6}\theta_1^3 - \frac{1}{2}\theta_1^2\beta}{2\lambda_1 + \theta_1 - \beta}}{\tau_{Ip}} - \frac{\lambda_1^2 + \theta_1\beta - \frac{1}{2}\theta_1^2}{2\lambda_1 + \theta_1 - \beta}$	$\tau_{Dp} = \frac{\frac{1}{2}\theta_1^2}{\lambda_1 + \theta_1} + \frac{\tau_2^2 + \tau_1\lambda_2 - \lambda_2\tau_2 - \tau_2\tau_1 - \frac{\frac{1}{6}\theta_1^3}{\lambda_1 + \theta_1}}{\tau_{Ip}}$
Set-point filter	$q_{f1} = \frac{1}{\beta s + 1}; \quad \beta = \tau_1\left(1 - \left(1 - \frac{\lambda_1}{\tau_1}\right)^2 e^{\frac{-\theta_1}{\tau_1}}\right)$	$q_{f1} = 1$

[a] In case A, a PID·lag controller is always used, the lag filter is set as $\frac{1}{\tau_2 s + 1}$.
[b] In case B, when a PID·lag controller is used, the lag filter is set as $\frac{1}{\tau_2 s + 1}$. The tuning rule is then modified simply by setting $\tau_2 = 0$.

which we factor into

$$p_{pm} = \frac{K_1(\tau_2 s+1)}{K_2(\lambda_2 s+1)^{r2}(\tau_1 s+1)}; \quad p_{pa} = e^{-\theta_1 s} \tag{41}$$

When the time constant τ_2 in the lead term is large (dominant), the PID · lag design is recommended for the primary controller. As mentioned earlier, a reasonable tuning for the PID · lag controller is obtained by using Eq. (28) for the process model with the ignored lead term [8], and then setting τ_F equal to τ_2.

Consider the case A (i.e., the desired closed-loop response of the primary loop is much faster than the open-loop system). To obtain a compact tuning rule, we utilize the simplified method for the PID · lag controller design by ignoring τ_2. Furthermore, since the secondary loop is typically tuned to be sufficiently faster than the primary loop (i.e., $\lambda_2 \ll \lambda_1 \ll \tau_1$), Eq. (41) is simplified for controller design as

$$p_{pm} = \frac{K_1}{K_2(\tau_1 s+1)} \tag{42}$$

The IMC filter is set to

$$f_1 = \frac{\beta s+1}{(\lambda_1 s+1)^2} \tag{43}$$

Therefore, the equivalent feedback controller of the primary loop becomes

$$G_{cp} = \frac{K_2(\tau_1 s+1)(\beta s+1)}{K_1[(\lambda_1 s+1)^2 - e^{-\theta_1 s}(\beta s+1)]} \tag{44}$$

In the case B, which often occurs when the primary output is measured with a long time delay, the IMC filter is simply set to

$$f_1 = \frac{1}{(\lambda_1 s+1)} \tag{45}$$

and the equivalent feedback controller for the primary loop is equal to

$$G_{cp} = \frac{K_2(\lambda_2 s+1)(\tau_1 s+1)}{K_1(\tau_2 s+1)(\lambda_1 s+1-e^{-\theta_1 s})}. \tag{46}$$

When a PID · lag controller is used, the equivalent feedback controller is modified simply by setting $\tau_2 = 0$ in Eq. (46).

The PID controller, which approximates G_{cp} given by Eq. (44) or (46), is obtained using the tuning parameters given by Eq. (28). The result in the analytical form is listed in Table 1. The proposed tuning rules depend on λ_1 and λ_2, which are the desired closed-loop time constants of the primary and the secondary loops. The selection of λ_1 and λ_2, which control the tradeoff between robustness to plant-model mismatch and the achievable closed-loop performance, may require several iterations. In process control practice, the closed-loop time constants are rarely selected to be smaller than a tenth of the dominant open-loop time constant. The selection of λ should also depend on process time delays. Our experience suggests the following initial choice of λ: For the case A, $\lambda_1/\theta_1 \geqslant 1$ and $\lambda_2/\theta_2 \geqslant 1$; For the case B, $\lambda_1/\theta_1 \geqslant 0.5$ and $\lambda_2/\theta_2 \geqslant 0.5$.

5. Simulation study

Example 1. Consider the process [1] with

$$p_1 = p_{d1} = \frac{e^{-4s}}{(20s+1)}$$

$$p_2 = p_{d2} = \frac{1}{(10s+1)}$$

Though both the primary and the secondary processes are lag dominant, we design PID controllers using rules for both cases A and B and compare their performance. The PID · lag controller is used as a primary controller because the time constant of the secondary process is significant. We select $\lambda_2 = 1$ and $\lambda_1 = 4$. The resulting tuning parameters are listed in Table 2.

During computer simulations, the unit step disturbance is applied at $t = 0$. The closed-loop response of the primary output is shown in Fig. 5 with the IMC controllers for cases A and B, and their PID approximations. The response with the design developed in [1] is also shown. As expected, the IMC controller based on case A design shows the best performance. When approximated by the PID controller, the response becomes more oscillatory but still better than the alternatives. The responses with the IMC controller for the case B design and its PID approximation are essentially identical. Note that performance of the PID controller with

Table 2
PID tuning values for Example 1

	Secondary loop				Primary loop				
	K_{cs}	τ_{Is}	τ_{Ds}	q_{f2}	K_{cp}	τ_{Ip}	τ_{Dp}	Lag	q_{f1}
Yu	1	10	0		3.24	22	0		
Proposed (case B)	10	10	0		2.75	22	1.85	$\frac{1}{10s+1}$	
Proposed (case A)	19	1.9	0	$\frac{1}{1.9s+1}$	4.41	10.9	1.24	$\frac{1}{10s+1}$	$\frac{1}{9.52s+1}$

1DOF parallel cascade structure (i.e., case B) is still superior to the alternative design.

The quality of the PID approximations of the equivalent feedback controllers was also characterized in the frequency domain. Fig. 6 shows that for the case B design, the approximation is excellent over the entire range of frequencies. The case A design leads to a more complex form of the IMC controller, which can be accurately approximated by the PID controller only at low frequencies.

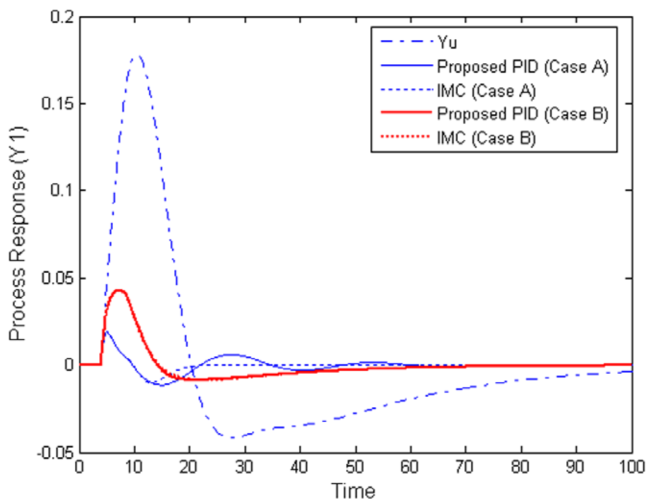

Fig. 5. Example 1: Comparison of the closed-loop responses to a load change.

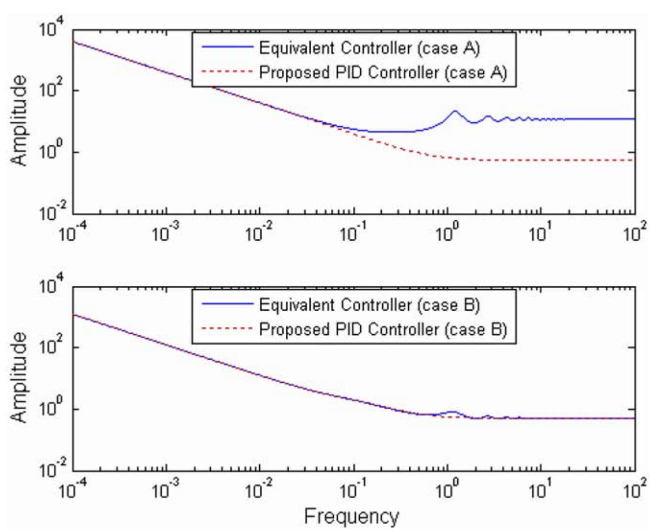

Fig. 6. Comparison of frequency plots for the equivalent feedback controller and the proposed PID controllers for Example 1.

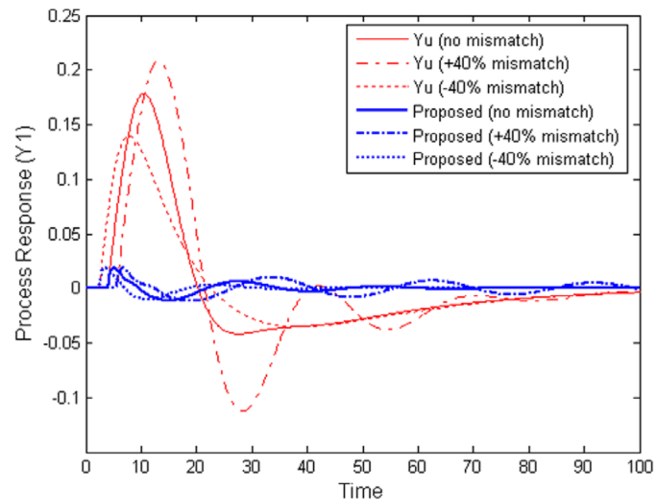

Fig. 7. Example 1: Comparison of the closed-loop responses to a load change under process model uncertainty.

To investigate robustness of the proposed controller, we suppose that there exists ±40% error for estimating process model parameters in both the primary and secondary loops. For instance, the process gain, the time constant and the time delay are actually 40% larger (or smaller) than those in the nominal model. The closed-loop response by the proposed controller (case A) is provided in Fig. 7 in comparison with that by [1]. Fig. 7 shows that the proposed controller holds the control system robust stability as well in the presence of the severe process uncertainty.

Example 2. The following process model was used by Semino and Brambilla [5]:

$$p_1 = p_{d1} = \frac{1.24e^{-33s}}{(30s+1)}$$

$$p_2 = p_{d2} = \frac{3.1e^{-9s}}{(30s+1)}$$

The closed-loop time constants are selected as $\lambda_2 = 5$ and $\lambda_1 = 17$. The secondary process is clearly lag-time dominant and thus belongs to case A. The primary pro-

Table 3
PID tuning values for Example 2

	Secondary loop				Primary loop				
	K_{cs}	τ_{Is}	τ_{Ds}	q_{f2}	K_{cp}	τ_{Ip}	τ_{Dp}	Lag	q_{f1}
Semino and Brambilla	0.77	34.5	0		0.78	12	0		
Proposed (case 1)	0.76	32.9	2.63		2.30	45.9	11.6	$\frac{1}{30s+1}$	
Proposed (case 2)	1.35	18.5	3.27	$\frac{1}{14.6s+1}$	2.30	45.9	11.6	$\frac{1}{30s+1}$	
Proposed (case 3)	1.35	18.5	3.27	$\frac{1}{14.6s+1}$	2.63	40.8	9.24	$\frac{1}{30s+1}$	$\frac{1}{28.1s+1}$

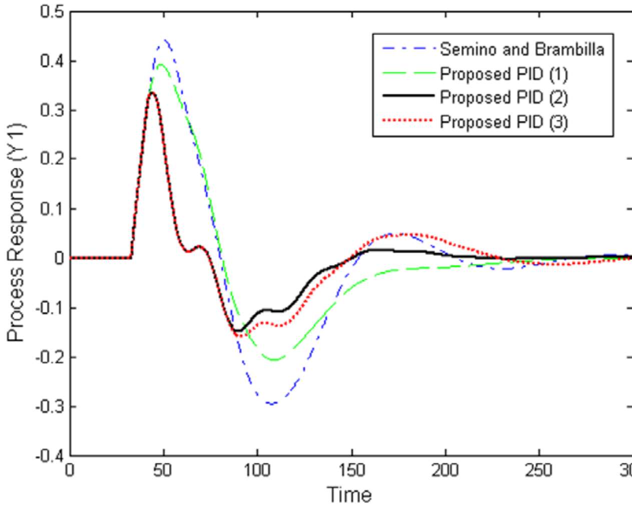

Fig. 8. Example 2: Comparison of the closed-loop responses to a load change.

cess is dead-time dominant and belongs to case B. Because of significant dead time, the desired closed-loop dynamics of the primary loop should not be selected sufficiently faster than the open-loop dynamics. It is, therefore, expected that the cancellation of process pole will give little benefit, but will result in a more complex controller.

The design of the parallel cascade controllers was carried out for the following three cases: (1) both the primary and secondary controllers are designed using the procedure for case B; (2) the case A design is used for the primary loop, while the secondary controller is designed using the case B procedure; (3) the case A design is used to obtain, both, primary and secondary controllers. The result for all three cases is summarized in Table 3, and compared by computer simulations for the closed-loop response to a unit change in the load disturbance applied at $t = 0$. Fig. 8 shows the results for all designs, and compares them with the response obtained with the controller described in [5]. As expected, the PID design for the case (2) gives the best performance and all our PID controller designs outperform the alternative.

Example 3. Consider the following GPL splitter model [9]:

$$p_1 = \frac{-0.0067e^{-20s}}{105.8s+1}; \quad p_{d1} = \frac{0.05843e^{-20s}}{115.5s+1}$$
$$p_2 = \frac{-5.217}{101.6s+1}; \quad p_{d2} = \frac{44.15}{109.5s+1}$$

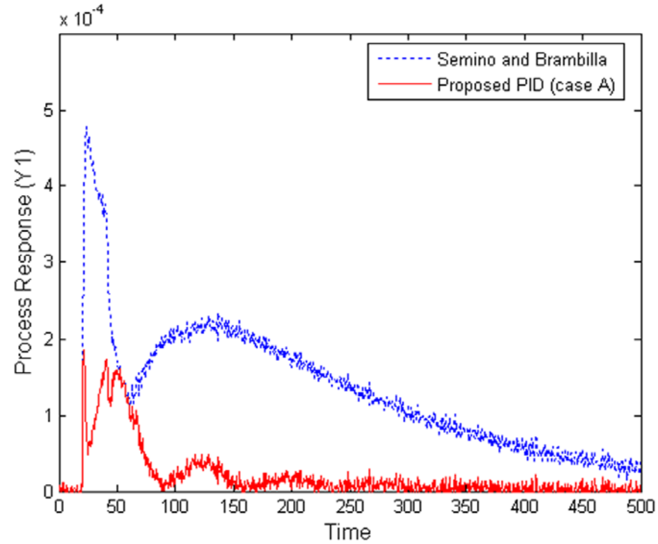

Fig. 9. Example 3: Comparison of the closed-loop responses to a load change.

Since the process time constants are significantly larger than the time delay, the process should be classified as case A. The closed-loop time constants are chosen as $\lambda_2 = 1$ and $\lambda_1 = 10$. The resulting tuning parameters are listed in Table 4. During simulation, a unit step change in the load disturbance has been introduced at $t = 0$. White noises are added to the primary output to reflect the noise effect from real process measurements. Note that a difference in the pole location in p and p_d causes a mismatch in pole-zero cancellation when tuning rules of Table 1 are used to design the controllers. Fig. 9 compares the primary output response with the proposed controller and the controller designed in [5], and shows a considerably improved performance achieved with the developed approach.

6. Conclusions

We have developed an approach to the design of the parallel cascade control systems which improve setpoint tracking and disturbance rejection. Both primary and secondary loops are controlled by 2DOF controllers, which are designed following the IMC paradigm. The details of the IMC designs for two typical cases (with or without cancellation of a pole in the process model) are elaborated.

The designed IMC controllers may be complex and high dimensional. Their PID approximation, following the

Table 4
PID tuning values for Example 3

	Secondary loop				Primary loop				
	K_{cs}	τ_{Is}	τ_{Ds}	q_{f2}	K_{cp}	τ_{Ip}	τ_{Dp}	Lag	q_{f1}
Semino and Brambilla	−19.5	20.3	0		195	10	0		
Proposed (case A)	−38.8	1.99	0	$\frac{1}{1.99s+1}$	5.6 E 3	43.2	7.73	$\frac{1}{101.6s+1}$	$\frac{1}{34s+1}$

approach described in [8], simplifies the practical implementation of the proposed approach at the expense of often minor performance degradation. Analytical tuning rules for the primary and the secondary PID controller are derived to approximate ideal feedback controllers designed using IMC approach. Using several examples, it is shown that the resulting PID controllers give an accurate approximation of IMC designs in, at least, low frequency range.

The main advantages of the proposed method are: (a) its ability to simultaneously improve setpoint tracking and disturbance rejection, which leads to a marked improvement in the closed-loop performance of the primary loop; (b) no restrictions on the form of the process model are placed, which implies broad applicability of the method; (c) simple analytical tuning rules are developed, which simplifies practical implementation of the parallel cascade systems in process industries; and (d) the method allows a simultaneous tuning of primary and secondary control loops.

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