Journal of Chemical Engineering of Japan, Vol. 40, No. 6, pp. 000–000, 2007 **Research Paper**

An Enhanced Performance PID Filter Controller for First Order Time Delay Processes

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Keywords: PID Controller Tuning, First Order Plus Dead Time Process, Disturbance Rejection, First Order Lead/Lag Filter, Two-Degree-of-Freedom Controller

An analytical tuning method for a PID controller cascaded with a lead/lag filter is proposed for FOPDT processes based on the IMC design principle. The controller is designed for the rejection of disturbances and a two-degree-of-freedom control structure is used to slacken the overshoot in the set-point response. The simulation study shows that the proposed design method provides better disturbance rejection than the conventional PID design methods when the controllers are tuned to have the same degrees of robustness. A guideline of a single tuning parameter of closed-loop time constant (λ) is provided for several different robustness levels.

Introduction

Proportional integral derivative (PID) controllers have been the most popular and widely used controllers in the process industries because of their simplicity, robustness and wide ranges of applicability with near-optimal performance. However, it has been noticed that many PID controllers are often poorly tuned and a certain amount of effort has been made to systematically resolve this problem.

The effectiveness of the internal model control (IMC) design principle has made it attractive in the process industries, where many attempts have been made to exploit the IMC principle to design PID controllers for both stable and unstable processes (Morari and Zafiriou, 1989). The IMC-PID tuning rules have the advantage of using only a single tuning parameter to achieve a clear trade-off between the closed-loop performance and robustness. The PID tuning methods proposed by Rivera *et al*. (1986), Morari and Zafiriou (1989), Horn *et al*. (1996), and Lee *et al*. (1998) are typical examples of the IMC-PID tuning method. The direct synthesis (DS) method proposed by Smith *et al*. (1975) and the direct synthesis for the disturbance (DS-d) method proposed by Chen and Seborg (2002) can also be categorized into the same class as the IMC-PID methods, in that they obtain the PID controller parameters by computing the ideal feedback controller which gives a predefined desired closed-loop response. Although the ideal controller is often more complicated than the PID controller for time delayed processes, the controller form can be reduced to that of either a PID controller or a PID controller cascaded with a low order filter by performing appropriate approximations of the dead time in the process model.

The control performance can be significantly enhanced by cascading the PID controller with a lead/lag filter, as given by Eq. (1).

$$G_c = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\frac{1+as}{1+bs} \tag{1}$$

where K_c, τ_I and τ_D are the proportional gain, integral time constant, and derivative time constant of the PID controller, respectively, and a and b are the filter parameters.

The structure of the PID controller cascaded with a filter was also suggested by Rivera *et al*. (1986), Morari and Zafiriou (1989), Horn *et al*. (1996), Lee *et al*. (1998) and Dwyer (2003). The PID filter controller in Eq. (1) can easily be implemented in modern control hardware.

It is essential to emphasize that the PID controller designed according to the IMC principle provides excellent set-point tracking, but has a sluggish disturbance response, especially for processes with a small time-delay/time-constant ratio (Morari and Zafiriou, 1989; Chien and Fruehauf, 1990; Horn *et al*., 1996; Lee *et al*., 1998; Chen and Seborg, 2002; Skogestad, 2003). Since disturbance rejection is much more important than set-point tracking for many process control applications, a controller design that emphasizes the former rather than the latter is an important design goal that has recently been the focus of renewed research.

Received on November 27, 2006; accepted on February 15, 2007. Correspondence concerning this article should be addressed to M. Lee (E-mail address: mynlee@yu.ac.kr).

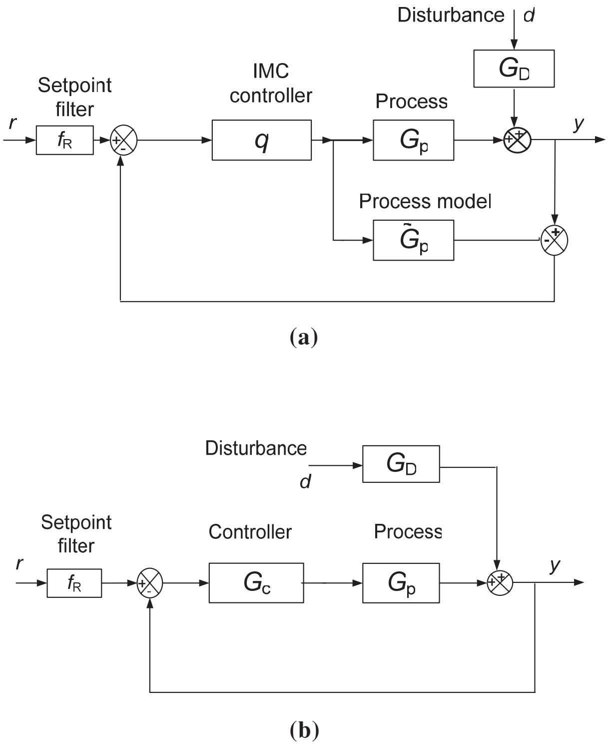

Fig. 1 Block diagram of IMC and classical feedback control systems: (a) The IMC structure; (b) Feedback control structure

In the present study, a simple and efficient method is proposed for the design of a PID filter controller with enhanced performance. A closed-loop time constant (λ) guideline is recommended for a wide range of time-delay/time-constant ratios. A simulation study was performed to illustrate the superiority of the proposed method for both nominal and perturbed processes.

1. IMC Controller Design Procedure

Figures 1(a) and **(b)** show the block diagrams of the IMC control and equivalent classical feedback control structures, respectively, where G_P is the process, $\tilde{G}_P$ the process model, q the IMC controller, f_R the set-point filter, and G_c the equivalent feedback controller.

For the nominal case (i.e., $G_P = \tilde{G}_P$), the set-point and disturbance responses in the IMC control structure can be simplified as:

$$y = G_P q r + \left(1 - \tilde{G}_P q\right) G_D d \tag{2}$$

According to the IMC parameterization (Morari and Zafiriou, 1989), the process model $\tilde{G}_P$ is factored into two parts:

$$\tilde{G}_P = p_m p_A \tag{3}$$

where p_m is the portion of the model inverted by the controller, p_A is the portion of the model not inverted by the controller and $p_A(0) = 1$. The noninvertible part usually includes the dead time and/or right half plane zeros and is chosen to be all-pass.

To obtain a good response for processes with poles near zero, the IMC controller q should be designed to satisfy the following conditions.

1. If the process G_P has poles near zero at $z_1, z_2, \ldots, z_m$, then q should have zeros at $z_1, z_2, \ldots, z_m$.
2. If the process G_D has poles near zero, $z_{d1}, z_{d2}, \ldots, z_{dm}$, then $(1 - G_P q)$ should have zeros at $z_{d1}, z_{d2}, \ldots, z_{dm}$.

Since the IMC controller q is designed as $q = p_m^{-1} f$, the first condition is satisfied automatically. The second condition can be fulfilled by designing the IMC filter f as

$$f = \frac{\sum_{i=1}^{m} \beta_i s^i + 1}{(\lambda s + 1)^r} \tag{4}$$

where λ is an adjustable parameter which controls the tradeoff between the performance and robustness; r is selected to be large enough to make the IMC controller (semi-)proper; β_i are determined by Eq. (5) to cancel the poles near zero in G_D.

$$1 - G_P q\big|_{s=z_{d1},\cdots,z_{dm}} = \left| 1 - \frac{p_A\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)}{(\lambda s + 1)^r} \right|_{s=z_{d1},\cdots,z_{dm}} = 0 \tag{5}$$

Then, the IMC controller comes to be

$$q = p_m^{-1} \frac{\sum_{i=1}^{m} \beta_i s^i + 1}{(\lambda s + 1)^r} \tag{6}$$

Thus, the closed-loop response is

$$y = \frac{p_A\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)}{(\lambda s + 1)^r} r + \left(1 - \frac{p_A\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)}{(\lambda s + 1)^r}\right) G_D d \tag{7}$$

From the above design procedure, one can achieve a stable closed-loop response by using the IMC controller.

2. PID filter Controller Design for FOPDT Process

The ideal feedback controller that is equivalent to the IMC controller can be expressed in terms of the internal model $\tilde{G}_P$ and the IMC controller q:

$$G_c = \frac{q}{1-\tilde{G}_P q} \tag{8}$$

Substituting Eqs. (3) and (6) into Eq. (8) gives the ideal feedback controller:

$$G_c = \frac{p_m^{-1}\dfrac{\sum_{i=1}^{m}\beta_i s^i + 1}{(\lambda s+1)^r}}{1-\dfrac{p_A\left(\sum_{i=1}^{m}\beta_i s^i + 1\right)}{(\lambda s+1)^r}} \tag{9}$$

Let us consider the first order plus dead time (FOPDT) process, which is most widely utilized in the chemical process industries, as a representative model.

$$G_P = G_D = \frac{Ke^{-\theta s}}{\tau s+1} \tag{10}$$

where K is the gain, τ the time constant, and θ the time delay. The IMC filter structure is

$$f = \frac{\beta s+1}{(\lambda s+1)^2} \tag{11}$$

It is noticed that the IMC filter form in Eq. (11) was also utilized by Lee *et al*. (1998) and Horn *et al*. (1996). The resulting IMC controller becomes

$$q = \frac{(\tau s+1)(\beta s+1)}{K(\lambda s+1)^2} \tag{12}$$

Therefore, the ideal feedback controller is obtained as

$$G_c = \frac{(\tau s+1)(\beta s+1)}{K\left[(\lambda s+1)^2 - e^{-\theta s}(\beta s+1)\right]} \tag{13}$$

Since the ideal feedback controller in Eq. (13) does not have the PID filter controller form, the remaining issue is how to design the PID filter controller that approximates the ideal feedback controller most closely.

Approximating the dead time $e^{-\theta s}$ with a 2/2 Pade expansion

$$e^{-\theta s} = \frac{1-\dfrac{\theta s}{2}+\dfrac{\theta^2 s^2}{12}}{1+\dfrac{\theta s}{2}+\dfrac{\theta^2 s^2}{12}} \tag{14}$$

results in G_C as

$$G_c = \frac{(\tau s+1)\left(1+\dfrac{\theta s}{2}+\dfrac{\theta^2 s^2}{12}\right)(\beta s+1)}{K\left[(\lambda s+1)^2\left(1+\dfrac{\theta s}{2}+\dfrac{\theta^2 s^2}{12}\right)-(\beta s+1)\left(1-\dfrac{\theta s}{2}+\dfrac{\theta^2 s^2}{12}\right)\right]} \tag{15}$$

It is important to note that the 2/2 Pade approximation is precise enough to convert the ideal feedback controller into a finite dimensional feedback controller with barely any loss of accuracy. Expanding and rearranging Eq. (15) gives

$$G_c = \left(1+\frac{\theta s}{2}+\frac{\theta^2 s^2}{12}\right)(\tau s+1)(\beta s+1) \Bigg/ K(2\lambda-\beta+\theta)\left[1+\left(\frac{\dfrac{\beta\theta}{2}+\lambda\theta+\lambda^2}{2\lambda-\beta+\theta}\right)s + \left(\frac{\dfrac{\beta\theta^2}{12}+\dfrac{\lambda\theta^2}{6}+\dfrac{\lambda^2\theta}{2}}{2\lambda-\beta+\theta}\right)s^2+\left(\frac{\dfrac{\lambda^2\theta^2}{12}}{2\lambda-\beta+\theta}\right)s^3\right] \tag{16}$$

As seen in Eq. (16), the resulting controller has the form of the PID controller cascaded with a high order filter. The analytical PID formula can be obtained as

$$K_c = \frac{\theta}{2K(2\lambda-\beta+\theta)},\quad \tau_I = \frac{\theta}{2},\quad \tau_D = \frac{\theta}{6} \tag{17}$$

The value of the extra degree of freedom β is selected so that it cancels out the open-loop pole at $s = -1/\tau$ that causes a sluggish response to load disturbances. From Eq. (5), this requires $[1-(\beta s+1)e^{-\theta s}/(\lambda s+1)^2]|_{s=-1/\tau} = 0$. Thus, the value of β is obtained as

$$\beta = \tau\left[1-\left(1-\frac{\lambda}{\tau}\right)^2 e^{-\theta/\tau}\right] \tag{18}$$

Furthermore, it is obvious from Eq. (5) that the remaining part of the denominator in Eq. (16) contains the factor $(\tau s + 1)$. Therefore, the filter parameter b in Eq. (1) can be obtained by taking the first derivative of Eq. (19) below

$$cs^2+bs+1 = \frac{1+\left(\dfrac{\dfrac{\beta\theta}{2}+\lambda\theta+\lambda^2}{2\lambda-\beta+\theta}\right)s+\left(\dfrac{\dfrac{\beta\theta^2}{12}+\dfrac{\lambda\theta^2}{6}+\dfrac{\lambda^2\theta}{2}}{2\lambda-\beta+\theta}\right)s^2+\left(\dfrac{\dfrac{\lambda^2\theta^2}{12}}{2\lambda-\beta+\theta}\right)s^3}{\tau s+1} \tag{19}$$

and substituting $s = 0$ as

$$b = \frac{\frac{\beta\theta}{2} + \lambda\theta + \lambda^2}{2\lambda - \beta + \theta} - \tau \tag{20}$$

The filter parameter in Eq. (1) can be easily obtained from Eq. (16) as

$$a = \beta \tag{21}$$

Since the high order cs^2 term has little impact on the overall control performance in the control relevant frequency range, the remaining part of the fraction in Eq. (16) can be successfully approximated to a simple first order lead/lag filter as $(1 + as)/(1 + bs)$. Our simulation result (although not shown in this paper) also confirms the validity of this model reduction.

The lead term $(\beta s + 1)$ in the closed-loop transfer function of Eq. (7) causes excessive overshoot in the set-point response, which can be eradicated by adding the set-point filter f_R as:

$$f_R = \frac{\gamma\beta s + 1}{\beta s + 1} \tag{22}$$

where $0 \le \gamma \le 1$. The extreme case with $\gamma = 0$ has no lead term in the set-point filter which would cause a slow servo response. On the other hand, $\gamma = 1$ means that there is no set-point filter. γ can be adjusted online to obtain the desired speed of the set-point response. The proposed study is also applicable to the process with negligible dead time while it is mainly focused on the first order time delay process.

3. Robust Stability

The well-known robust stability theorem can be utilized to analyze the robust stability of the proposed controller.

Robust Stability Theorem (Morari and Zafirou, 1989): Let us assume that all plants G_P in the family Π

$$\Pi = \left\{ G_p : \left| \frac{G_p(i\omega) - \tilde{G}_p(i\omega)}{\tilde{G}_p(i\omega)} \right| < \bar{\ell}_m(\omega) \right\} \tag{23}$$

have the same number of RHP poles and that a particular controller G_c stabilizes the nominal plant $\tilde{G}_p$. Then, the system is robustly stable with the controller G_c if and only if the complementary sensitivity function $\tilde{\eta}$ for the nominal plant $\tilde{G}_p$ satisfies the following bound:

$$\left\| \tilde{\eta}\bar{\ell}_m \right\|_\infty = \sup_\omega \left| \tilde{\eta}\bar{\ell}_m(\omega) \right| < 1 \tag{24}$$

Since $\tilde{\eta} = \tilde{G}_p q = \tilde{G}_p \tilde{p}_m^{-1} f$ for the IMC controller, the resulting Eq. (24) becomes:

$$\left| \tilde{G}_p \tilde{p}_m^{-1} f \bar{\ell}_m(\omega) \right| < 1 \quad \forall\, \omega \tag{25}$$

Thus, the above theorem can be interpreted as $|\bar{\ell}_m| < 1/|\tilde{\eta}| = 1/|\tilde{G}_p \tilde{p}_m^{-1} f|$, which guarantees robust stability when the multiplicative model error is bounded by $|\Delta_m(s)| \le \bar{\ell}_m$.

$$\left\| \tilde{\eta}(s)\Delta_m(s) \right\|_\infty < 1 \tag{26}$$

where $\Delta_m(s)$ defines the process multiplicative uncertainty bound. i.e., $\Delta_m(s) = (G_p - \tilde{G}_p)/\tilde{G}_p$. This uncertainty bound can be utilized to represent the model reduction error, process input actuator uncertainty, and process output sensor uncertainty, etc., which are very frequent in the actual process plants.

For the FOPDT process, the complementary sensitivity function $\tilde{\eta}(s)$ can be obtained as

$$\tilde{\eta}(s) = \frac{(\beta s + 1)e^{-\theta s}}{(\lambda s + 1)^2} \tag{27}$$

Substituting Eq. (27) and β into Eq. (26) yields the robust stability constraint required for tuning the adjustable parameters λ

$$\left\| \frac{\tau\left[1 - \left(1 - \frac{\lambda}{\tau}\right)^2 e^{-\theta/\tau}\right]s + 1}{(\lambda s + 1)^2} \right\|_\infty < \frac{1}{\left\| \Delta_m(s) \right\|_\infty} \tag{28}$$

Substituting $s = i\omega$ into Eq. (28) results in

$$\left\| \frac{\sqrt{\tau^2\left[1 - \left(1 - \frac{\lambda}{\tau}\right)^2 e^{-\theta/\tau}\right]^2 \omega^2 + 1}}{\lambda^2\omega^2 + 1} \right\|_\infty < \frac{1}{\left\| \Delta_m(\iota\omega) \right\|_\infty} \tag{29}$$

It is possible for uncertainty to occur in any of the three process parameters i.e., θ, τ, and K. Consequently, we have to consider the uncertainty in the different parameters separately. Let us consider the FOPDT process having the uncertainty in all three parameters as

$$G_p = \frac{(K + \Delta K)e^{-(\theta + \Delta\theta)s}}{(\tau s + 1)(\Delta\tau s + 1)} \tag{30}$$

It is most common practice that the FOPDT model approximated from the high order process in the real process plant. Due to this for the time constant uncertainty it is assumed that the small time constant $\Delta\tau$ is neglected/missing in developing the nominal model as considered in Eq. (30) (Seborg *et al*., 2004). Then the process multiplicative uncertainty bound becomes

$$\Delta_{\mathrm{m}}(s) = \frac{\left(1+\dfrac{\Delta K}{K}\right)e^{-\Delta\theta s}}{\Delta\tau s+1} - 1 \tag{31}$$

Substituting the above result into Eq. (29), we obtain the robust stability constraint as follows:

$$\frac{\sqrt{\tau^2\left[1-\left(1-\dfrac{\lambda}{\tau}\right)^2 e^{-\theta/\tau}\right]^2 \omega^2+1}}{\lambda^2\omega^2+1} < 1\Bigg/\left|\frac{\left(1+\dfrac{\Delta K}{K}\right)e^{-\Delta\theta i\omega}}{\Delta\tau i\omega+1}-1\right|,$$
$$\forall\omega > 0 \tag{32}$$

The above robust stability constraint is very useful to adjust λ where there is uncertainty in the process parameters. The robust stability constraint in Eq. (32) can also be used to determine the maximum allowable values of uncertainty in $\pm\Delta K$, $\pm\Delta\theta$ and $\pm\Delta\tau$ or various combinations of them for which robust stability can be guaranteed. For example, a plot of $|\tilde{\eta}(\omega)\,\bar{\ell}_m(\omega)|$ vs. ω can be constructed for a small value of any parametric uncertainty and/or combination of different uncertainties.

4. Simulation Study

This section deals with the simulation study conducted for three representative FOPDT processes: the lag time dominant process, the equal dead time and lag time process, and the dead time dominant process.

To evaluate the robustness of a control system, the maximum sensitivity, *Ms*, which is defined by $Ms = \max|1/[1 + G_pG_c(i\omega)]|$, is used. Since the *Ms* is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point (–1, 0), a small *Ms* value indicates that the stability margin of the control system is large. The *Ms* is a well-known robustness measure and is used by many researchers (Skogestad and Postlethwaite, 1996; Åström *et al*., 1998; Chen and Seborg, 2002; Skogestad, 2003).

Typical values of *Ms* are in the range of 1.2–2.0 (Åström *et al*., 1998; Seborg *et al*., 2004). To ensure a fair comparison, it is widely accepted for the model-based controllers (DS-d, DS, and IMC) to tune by adjusting λ so that the *Ms* values become the same values. Therefore, throughout all our simulation examples, all of the controllers compared were designed to have the same robustness level in terms of the maximum sensitivity, *Ms*.

To evaluate the closed-loop performance, two performance indices were considered in the case of both a step set-point change and a step load disturbance, viz., the integral of the time-weighted absolute error (ITAE) defined by $ITAE = \int_0^\infty t|e(t)|\mathrm{d}t$, and the overshoot which acts as a measure of how much the response exceeds the ultimate value following a step change in the set-point and/or disturbance.

In this paper, the simulation study has been conducted using the PID controller in the form of Eq. (1). However, for real implementation, the "parallel form" of the PID controller, $G(s) = K_c\{1 + 1/(\tau_I s) + \tau_D s/[(0.1\tau_D s) + 1]\}(1 + as)/(1 + bs)$, which is widely used in the real processes, can be applied to approximately the same performance.

To evaluate the usage of manipulated input values, we compute *TV* of the input $u(t)$, which is the sum of all of its movement of up and down. If we discretize the input signal as a sequence $[u_1, u_2, u_3, \ldots, u_i, \ldots]$, then $TV = \sum_{i=1}^{\infty}|u_{i+1} - u_i|$ should be as small as possible. *TV* is a good measure of the smoothness of a signal (Skogestad and Postlethwaite, 1996; Chen and Seborg, 2002; Skogestad, 2003).

4.1 Example 1: Lag time dominant process (θ/τ = 0.01)

Consider the following FOPDT process (Chen and Seborg, 2002; Seborg *et al*., 2004):

$$G_{\mathrm{p}} = G_{\mathrm{D}} = \frac{100e^{-1s}}{100s+1} \tag{33}$$

The proposed PID filter controller is compared with other controllers based on existing methods, such as the DS-d method, and those proposed by Rivera *et al*. (1986), Horn *et al*. (1996), Lee *et al*. (1998) and Lee *et al*. (1998) with a conventional filter. The controller parameters, including the performance and robustness matrix, are listed in **Table 1**. In order to ensure a fair comparison, all of the controllers compared are tuned to have *Ms* = 1.94 by adjusting λ. **Figure 2** compares the set-point and load responses obtained using the proposed method, the DS-d method, and the methods proposed by Lee *et al*. (1998) and Horn *et al*. (1996). The 2DOF controller using the set-point filter was used in the DS-d method and the methods proposed by Lee *et al*. (1998) and Horn *et al*. (1996) to obtain an enhanced set-point response. It is important to note that the set-point filter used for the set-point response has a clear benefit when the process is lag time dominant. In this case, it is observed that $0.4 \le \gamma \le 0$ gives smooth and robust control performances. In the proposed controller, γ in the set-point filter is selected as $\gamma = 0.45$. The closed-loop response for both the set-point tracking and disturbance rejection signifies that the proposed method provides a superior response for the same robustness.

Table 1 PID controller parameters and performance matrix for example 1 ($\theta/\tau = 0.01$)

Tuning methods	λ	K_c	τ_I	τ_D	Set-point			Disturbance		
					ITAE	Overshoot	TV	ITAE	Overshoot	TV
Proposed method[a]	1.131	0.124	0.50	0.167	1.96	0.007	3.17	14.55	1.206	1.96
Lee *et al*. (1998)[b]	1.330	0.806	3.947	0.3068	8.46	0.0	0.542	19.77	1.314	1.79
DS-d[c]	1.202	0.826	4.059	0.353	3.13	0.015	0.87	20.43	1.273	1.88
Horn *et al*. (1996)[d]	1.689	15.038	100.50	0.497	12.45	0.0	0.430	31.18	1.478	1.69
Rivera *et al*. (1986)[e]	0.408	0.714	100.50	0.4975	3.86	0.025	1.47	3785.0	1.411	1.35
Lee *et al*. (1998)[f]	0.248	0.805	100.41	0.399	3.15	0.018	1.136	3354.0	1.273	1.53

[a] $G_c = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1+as}{1+bs}$, where $a = 3.222$, $b = 0.139$; $f_R = \frac{1.45s+1}{3.22s+1}$

[b] $f_R = \frac{1}{3.66s+1}$

[c] $f_R = \frac{2.03s+1}{1.43s^2+4.06+1}$

[d] $G_c = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1+as}{1+bs+cs^2}$, where $a = 4.311$, $b = 100.2$, $c = 21.34$; $f_R = \frac{1}{4.31s+1}$

[e] $G_c = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1}{1+bs}$, where $b = 0.145$

[f] The Lee *et al*. (1998) method based on the conventional IMC filter form of $f = 1/(\lambda s + 1)$

*1DOF controller is used only for the methods of Rivera *et al*. (1986)[e] and Lee *et al*. (1998)[f]

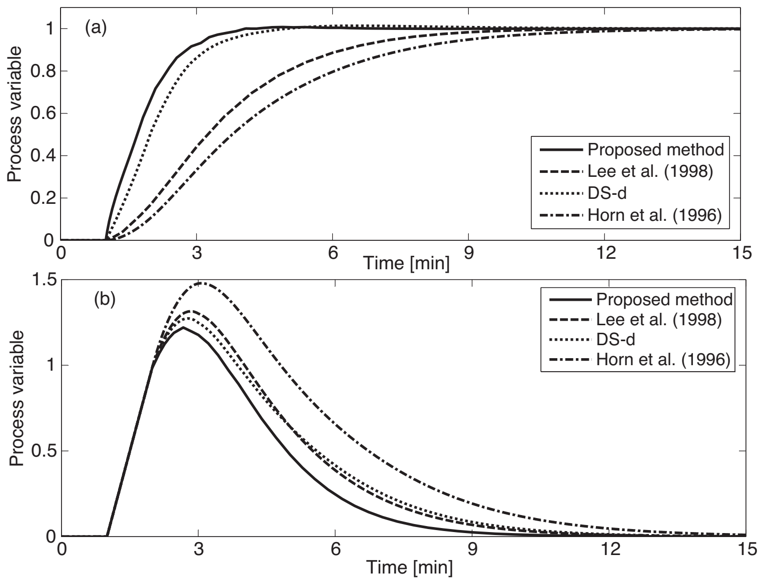

Fig. 2 Simulation results for example 1

The robust performance is evaluated by inserting a perturbation uncertainty of 20% in all three parameters in the worst direction simultaneously and finding the actual process as $G_p = G_D = 120e^{-1.2s}/(80s + 1)$. The simulation results for the model mismatch for various methods are given in **Table 2**. The performance and robustness indices obviously demonstrate that the proposed method has more robust performance than the others.

4.2 Example 2: Equal lag time and dead time process ($\theta/\tau = 1$)

Consider the process model described by Chen and Seborg (2002) as follows

$$G_p = G_D = \frac{1e^{-1s}}{1s+1} \tag{34}$$

Table 3 PID controller parameters and performance matrix for example 2 ($\theta/\tau = 1$)

Tuning methods	λ	K_c	τ_I	τ_D	Set-point			Disturbance		
					ITAE	Overshoot	TV	ITAE	Overshoot	TV
Proposed method[a]	0.499	0.458	0.5	0.166	2.28	0.0034	2.835	2.66	0.626	2.837
Lee *et al.* (1998)[b]	0.596	1.042	1.304	0.270	2.83	0.0025	1.050	3.31	0.622	1.368
DS-d[c]	0.771	1.055	1.444	0.313	2.71	0.0006	1.375	3.85	0.633	1.419
Horn *et al.* (1996)[d]	0.73	1.010	1.50	0.333	3.59	0.0002	1.059	4.12	0.672	1.163
Rivera *et al.* (1986)[e]	0.503	0.998	1.50	0.333	2.33	0.1481	2.124	4.26	0.670	1.151
Lee *et al.* (1998)[f]	0.309	1.055	1.382	0.289	1.95	0.1273	1.981	3.52	0.634	1.394

[a] $a = 0.907,\ b = 0.102;\ f_R = \dfrac{1}{0.908s+1}$

[b] $f_R = \dfrac{1}{0.94s+1}$

[c] $f_R = \dfrac{0.722s+1}{0.452s^2+1.44s+1}$

[d] $a = 0.975,\ b = 1.179,\ c = 0.179;\ f_R = \dfrac{1}{0.976s+1}$

[e] $b = 0.167$

*1DOF controller is used only for the methods of Rivera *et al.* (1986)[e] and Lee *et al.* (1998)[f]

Table 2 Robustness analysis for example 1

Tuning methods	Set-point		Disturbance	
	ITAE	Overshoot	ITAE	Overshoot
Proposed method[a]	13.36	0.3418	33.36	1.8798
Lee *et al.* (1998)[b]	14.40	0.0387	76.26	1.7399
DS-d[c]	40.41	0.2817	98.25	1.7011
Horn *et al.* (1996)[d]	16.32	0.0091	62.27	2.2096
Rivera *et al.* (1986)[e]	28.73	0.5354	3766.0	2.1152
Lee *et al.* (1998)[f]	21.21	0.5797	3338.0	1.9450

The proposed PID filter controller is compared with the DS-d controller and the controllers designed by Lee *et al.* (1998), Horn *et al.* (1996), Rivera *et al.* (1986) and Lee *et al.* (1998) with a conventional filter. The controller parameter values are listed in **Table 3** along with the performance matrix, where $Ms = 1.84$ is selected for all controller designs. Unit step changes are introduced both in the set-point and in the disturbance for the simulation. The simulation results in **Figure 3** indicate that both the disturbance and the set-point responses are faster in the proposed controller. The 2DOF controller structure is used for each design method except Rivera *et al.* (1986), and Lee *et al.* (1998) with a conventional filter. $\gamma = 0$ is selected for the proposed controller. It is clear from Figure 3 and Table 3 that the proposed controller exhibits better performance for both the set-point and disturbance response.

4.3 Example 3: Dead time dominant process ($\theta/\tau = 5$)

Consider the process with a long dead time studied by Luyben (2001) and Chen and Seborg (2002)

$$G_p = G_D = \frac{1e^{-5s}}{1s+1} \tag{35}$$

The proposed and aforementioned design methods are compared. The controller settings with the performance matrices are given in **Table 4**. All of the controllers are designed to have $Ms = 1.74$. Since in the case of a dead time dominant process, the 1DOF controller is sufficient to achieve satisfactory control performance, no set-point filter is used for any design method.

The set-point and load responses are shown in **Figure 4**. From this figure, it is apparent that the proposed controller and the one designed by Lee *et al.* (1998) with the conventional filter provide similar responses, while the DS-d and Horn *et al.* (1996) methods exhibit sluggish responses and take a long time to settle the response.

The proposed controller has excellent performance when the lag time dominates, but its performance becomes similar to that of the methods based on the conventional filter when the dead time dominates. When $\theta/\tau >> 1$, the filter time constant should be chosen as $\lambda \approx \theta >> \tau$ for the sake of closed-loop stability. Therefore, the process pole at $-1/\tau$ is not a dominant pole in the closed-loop system. Instead, the pole at $-1/\lambda$ determines the overall dynamics. Thus, introducing the lead term ($\beta s + 1$) into the IMC filter to compensate the process pole at $-1/\tau$ has little impact on the disturbance response.

Furthermore, the lead term usually increases the complexity of the IMC controller, which in turn degrades the performance of the resulting PID controller

Table 4 PID controller parameters and performance matrix for example 3 ($\theta/\tau = 5$)

Tuning methods	λ	K_c	τ_I	τ_D	Set-point			Disturbance		
					ITAE	Overshoot	TV	ITAE	Overshoot	TV
Proposed method[a]	1.408	0.366	2.5	0.833	29.95	0.0451	1.268	68.34	0.989	1.312
Lee *et al*. (1998)[b]	1.423	0.408	2.799	0.721	30.79	0.0641	1.044	67.67	0.992	1.116
DS-d[c]	2.706	0.316	2.555	0.053	37.57	0.0115	0.766	85.88	0.984	1.052
Horn *et al*. (1996)[d]	2.648	0.430	3.5	0.714	41.76	0.0502	1.022	87.01	0.993	1.061
Rivera *et al*. (1986)[e]	3.117	0.431	3.5	0.714	40.86	0.0436	0.996	87.06	0.992	1.048
Lee *et al*. (1998)[f]	1.798	0.417	2.838	0.759	30.66	0.0642	1.115	66.77	0.990	1.066

[a] $a = 0.998$, $b = 0.689$
[d] $a = 2.164$, $b = 3.155$, $c = 2.156$
[e] $b = 0.96$
*1DOF controller is used for all of the methods

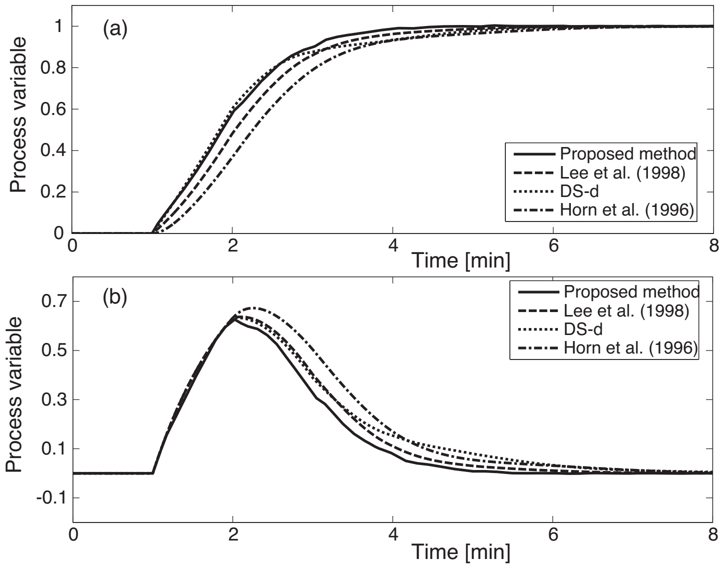

Fig. 3 Simulation results for example 2

by causing a larger discrepancy between the ideal feedback controller and thus the PID controller.

It is also important to note that as the order of the filter increases, the power of the denominator term ($\lambda s + 1$) also increases, which can cause an unnecessarily slow output response. As a result, in the case of a dead time dominant process, the PID controller based on the IMC filter that includes no lead term offers better performance.

4.4 Example 4: Polymerization process

An important viscosity loop in a polymerization process was identified by Chien *et al*. (2002) as follows:

$$G_p = G_D = \frac{3e^{-10s}}{100s+1} \tag{36}$$

The above-mentioned process has a large open-loop time constant of 100 min and a dead time of 10 min, which is also quite noteworthy. Chien *et al*. (2002) designed the PI controller with the modified Smith Predictor (SP) by approximating the above process in the form of an integrating model with a long dead time. **Figure 5** compares the nominal responses by the proposed PID filter controller and that by the modified SP. In the proposed controller, $\lambda = 8.0$ is selected and the resulting tuning parameters are obtained as $K_c = 0.6446$, $\tau_I = 5.0$, $\tau_D = 1.6667$, $a = 23.4146$ and $b = 0.9781$. The simulation was conducted by inserting the step set-point change at $t = 0$ followed by a load step change of -1.0 at $t = 90$.

The proposed controller uses a simple feedback control structure without any dead time compensator. Nevertheless, the proposed PID filter controller provides a superior performance, as shown in Figure 5.

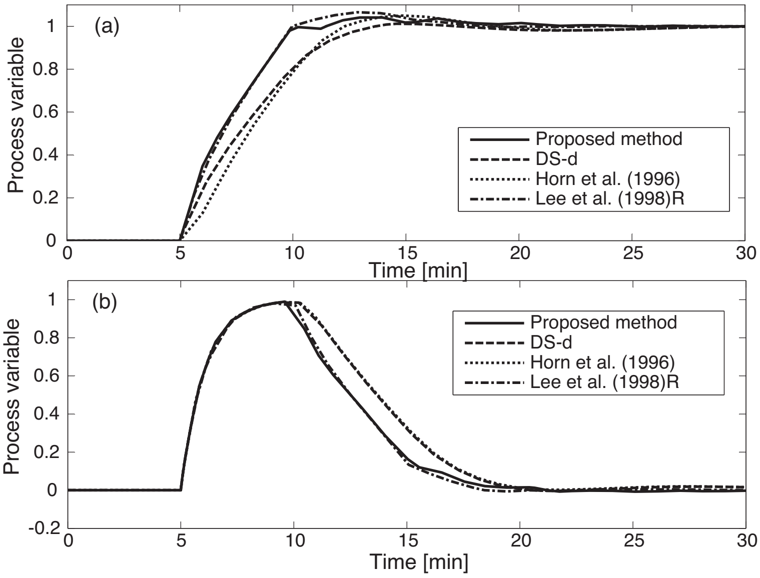

Fig. 4 Simulation results for example 3

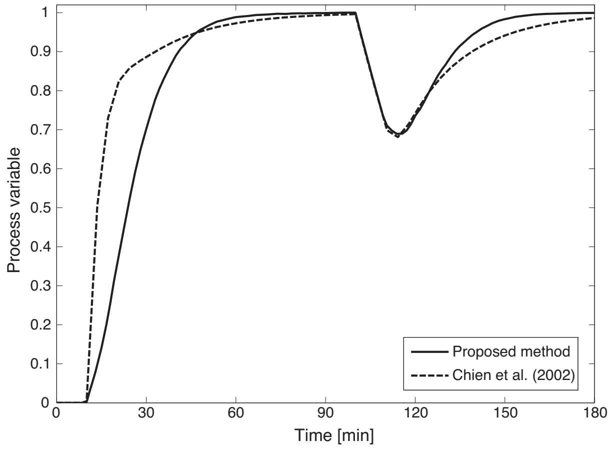

Fig. 5 Simulation results of the polymerization process

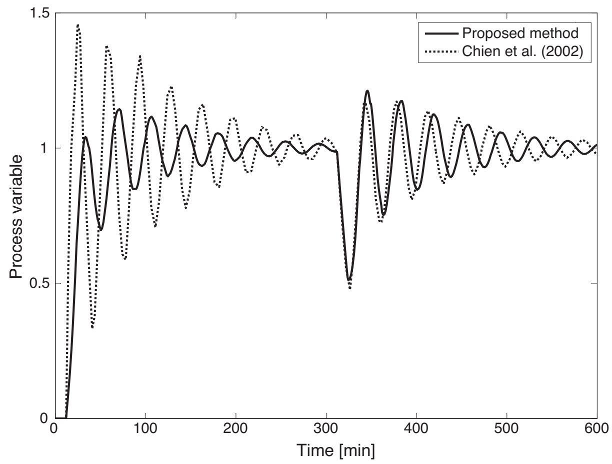

Fig. 6 Simulation results of the polymerization process with model mismatch

The disturbance rejection afforded by the proposed controller has a smaller settling time, whereas the modified SP controller described by Chien *et al*. (2002) shows a sluggish and required long settling time.

As regards the set-point response, the modified SP controller has an initially fast response, because of the elimination of the dead time, but afterwards it becomes slow. On the other hand, the speed of the response for the proposed controller is uniform and the settling time is similar to that by the modified SP.

It is important to note that the SP control configuration has a clear advantage of eliminating the time delay from the characteristic equation, which is very effective to set-point tracking performance. However, this advantage is lost if the process model is inaccurate. In order to evaluate the robustness against model uncertainty, a simulation study was conducted for the worst case of model mismatch by assuming that the process has a 20% mismatch in the three process parameters in the worst direction, as follows

$$G_{\mathrm{p}} = G_{\mathrm{D}} = \frac{3.6e^{-12s}}{80s+1} \tag{37}$$

The closed-loop responses are presented in **Figure 6**. Notice that the proposed method and the modified SP method described by Chien *et al*. (2002) have similar disturbance rejection responses for the model mismatch case. However, the set-point response afforded by the modified SP controller shows severe oscillation, while the proposed controller gives a more robust response.

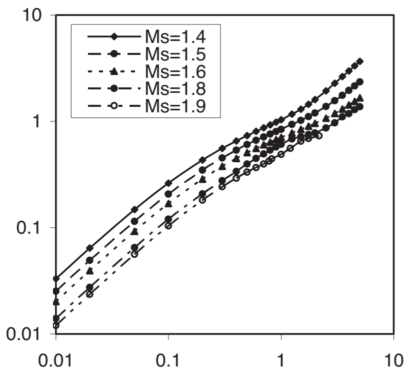

Fig. 7 λ guideline for the proposed tuning method

In the proposed tuning rule, the closed-loop time constant λ controls the tradeoff between the robustness and performance of the control system. As λ decreases, the closed-loop response becomes faster and can become unstable. On the other hand, as λ increases, the closed-loop response becomes stable but sluggish. A good tradeoff is obtained by choosing λ to give an Ms value in the range of 1.2–2.0 (Åström *et al.*, 1998; Seborg *et al.*, 2004). The λ guideline for several robustness levels is plotted in **Figure 7**.

Conclusions

A simple analytical design method for a PID controller cascaded with a lead/lag filter was proposed based on the IMC principle in order to improve its disturbance rejection performance. The proposed method also includes a set-point filter to enhance the set-point response like the 2DOF controller suggested by Lee *et al*. (1998), Horn *et al*. (1996) and Chen and Seborg (2002). FOPDT processes with three representative different θ/τ ratios were used for the simulation study. The proposed PID filter controller consistently provides superior performance over the whole range of the θ/τ ratio, while the other controllers based on the IMC-PID design methods take their advantage only in a limited range of the θ/τ ratio. In particular, the proposed controller shows excellent performance when the lag time dominates. The proposed controller was also compared with the more sophisticated controller, such as the modified Smith Predictor, in the case of the viscosity loop in a polymerization process. The result shows that the proposed controller gives satisfactory performance without the external dead time compensator. A guideline of closed-loop time constant λ was also proposed for a wide range of θ/τ ratio.

Acknowledgements

This research was supported by the 2006 Energy Resource and Technology Development Program of Korea.

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