

# Analytical design of enhanced PID filter controller for integrating and first order unstable processes with time delay

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## Abstract

An analytical design for a proportional-integral derivative (PID) controller cascaded with a first order lead/lag filter is proposed for integrating and first order unstable processes with time delay. The design algorithm is based on the internal model control (IMC) criterion, which has a single tuning parameter to adjust the performance and robustness of the controller. A setpoint filter is used to diminish the overshoot in the servo response. In the simulation study, the controllers were tuned to have the same degree of robustness by measuring the maximum sensitivity,  $M_s$ , in order to obtain a reasonable comparison. Furthermore, the robustness of the controller was investigated by inserting a perturbation uncertainty in all parameters simultaneously to obtain the worst case model mismatch, and the proposed method showed more robustness against process parameter uncertainty than the other methods. For the selection of the closed-loop time constant,  $\lambda$ , a guideline is also provided over a broad range of time-delay/time-constant ratios. The simulation results obtained for the suggested method were compared with those obtained for other recently published design methods to illustrate the superiority of the proposed method.

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## 1. Introduction

The proportional-integral-derivative (PID) control algorithm has three-term functionality enabling the treatment of both transient and steady-state responses; it provides a generic and efficient solution to real world control problems. The wide application of PID control has stimulated and sustained research and development in order to “get the best out of PID”, and the search is on to find the next key technology or methodology for PID tuning.

Many of the important chemical processing units in industrial and chemical practices are open-loop unstable processes that are known to be difficult to control, especially when there exists a time delay, such as in the case of continuous stirred tank reactors, polymerization reactors and bioreactors which are inherently open-loop unstable by design. Furthermore, many of these processes are usually run batch-wise, and as a result of

possible formulation changes, may operate with significant batch-to-batch variability. Clearly, the tuning of controllers to stabilize these processes and to impart adequate disturbance rejection is critical. Moreover, integrating processes are very frequently encountered in process industries and many researchers have suggested that for the purpose of designing a controller, considerable numbers of chemical processes could be modeled using an integrating process with time delay. Consequently, there has been much interest in the literature in the tuning of industrially standard PID controllers for open-loop unstable systems as well as for integrating processes.

The effectiveness of the internal model control (IMC) design principle has made this method attractive in the process industry, which has led to much effort being made to exploit the IMC principle to design equivalent feedback controllers for unstable processes (Morari and Zafriou, 1989). The IMC based PID tuning rules have the advantage of using only one tuning parameter to achieve a clear trade-off between closed-loop performance and robustness.

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It is well known that the IMC structure is very powerful for controlling stable processes with time delay and cannot be directly used for unstable processes because of the internal instability (Morari and Zafriou, 1989). For this reason, some modified IMC methods of two-degree-of-freedom (2DOF) control were developed for controlling unstable processes with time delay, such as those proposed by Lee et al. (2000), Yang et al. (2002), Wang and Cai (2002), Tan et al. (2003), and Liu et al. (2005). In addition, 2DOF control methods based on the Smith-Predictor (SP) were proposed by Majhi and Atherton (2000), Kwak et al. (1999), and Zhange et al. (2004) to achieve a smooth nominal setpoint response without overshoot for first order unstable processes with time delay. It is a notable merit of the modified IMC methods and the modified SP methods that the nominal setpoint response tends to be faster without overshoot for unstable processes. In fact, the common characteristic of the aforementioned modified IMC and SP methods is the use of a nominal process model in their control structures, which is responsible for their good performance in this respect. It should be noted that most existing 2DOF control methods are restricted to the unstable processes in the form of a first order rational part plus time delay, which in fact, cannot represent a variety of industrial and chemical unstable processes well enough. Besides, there usually exist unmodeled dynamics that inevitably tend to deteriorate the control system performance. The delay integrating process (DIP) has a clear advantage in the identification test, because the model contains only two parameters and is simple to use for identification. Some of the well-accepted PID controller tuning methods for DIPs are those proposed by Chien and Fruehauf (1990), Luyben (1996), and Chen and Seborg (2002). Shamsuzzoha et al. (2006) has suggested the IMC based design method of PID cascaded filter for the several types of processes. The design method mainly enhanced the setpoint response for the stable processes.

Modern control hardware provides the microprocessor implementation for a flexible combination of conventional control algorithms to achieve enhanced control performance. The PID controller cascaded with a first order lead/lag filter is a typical example. The main reason for using the PID-filter controller is that it provides better performance without tribulation. Earlier, many authors (Morari and Zafriou, 1989; Lee et al., 2000; Horn et al., 1996; Luyben, 1996; Shamsuzzoha et al., 2006) proposed the use of the PID controller cascaded with a first or second order filter, as described below

$$G_c = K_c \left( 1 + \frac{1}{\tau_I s} + \tau_D s \right) \frac{1 + as}{1 + bs}, \quad (1)$$

where  $K_c$ ,  $\tau_I$ , and  $\tau_D$  are the proportional gain, integral time constant, and derivative time constant of the PID controller, respectively, and  $a$  and  $b$  are the filter parameters. Recently, Rao and Chidambaram (2006) proposed a PID controller in series with a lead-lag compensator for the open-loop unstable, second order plus time delay processes with/without a zero. The method is based on direct synthesis and setpoint weighting is used to reduce the overshoot for servo response. Although their method has two tuning parameters, a significant improvement

is gained in load disturbance rejection performance for second order unstable process.

It should be emphasized that the design principle of the aforementioned tuning methods for unstable and integrating delay processes is complicated and that the modified IMC structure is difficult to implement in a real process plant in the presence of model uncertainty.

In this paper, a simple analytical method is proposed for the design of a PID-filter controller, in order to achieve enhanced performance for first order unstable and integrating delay processes. A closed-loop time constant,  $\lambda$ , guideline was recommended for a wide range of time-delay/time-constant ratios. A simulation study was performed for both unstable and integrating delay processes to show the superior performance of the proposed method for both nominal and perturbed processes.

## 2. Design procedure

The IMC controller is a competent method for control system design (Morari and Zafriou, 1989) as shown in Fig. 1, where  $G_p$  is the process transfer function,  $\tilde{G}_p$  the process model. Nevertheless, for unstable processes the IMC structure cannot be implemented because any bounded input,  $d$ , will produce unbound output,  $y$ , if  $G_p$  is unstable. As discussed in Morari and Zafriou (1989), the IMC approach to designing a controller for an unstable process is possible for  $G_p = \tilde{G}_p$  if the following conditions are satisfied for the internal stability of the closed-loop system:

- (i)  $q$  is stable.
- (ii)  $G_p q$  is stable.
- (iii)  $(1 - G_p q)G_p$  is stable.

These three conditions result in the well-known standard interpolation conditions (Morari and Zafriou, 1989):

- If the process model,  $G_p$ , has unstable poles,  $up_1, up_2, \dots, up_m$ , then  $q$  should have zeros at  $up_1, up_2, \dots, up_m$  and also  $1 - G_p q$  should have zeros at  $up_1, up_2, \dots, up_m$ .

Since the IMC controller,  $q$ , is designed as  $q = p_m^{-1} f$  in which  $p_m^{-1}$  includes the inverse of the model portion, the controller satisfies the first condition. The second condition could be satisfied through the design of the IMC filter,  $f$ . For this, the filter is designed as

$$f = \frac{\sum_{i=1}^m \alpha_i s^i + 1}{(\lambda s + 1)^r}, \quad (2)$$

where  $m$  is the number of poles to be canceled;  $\alpha_i$  are determined by Eq. (3) to cancel the unstable poles in  $G_p$ ;  $r$  is selected large enough to make the IMC controller proper.

$$1 - G_p q \Big|_{s=up_1, up_2, \dots, up_m} = \left| 1 - \frac{p_A (\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r} \right|_{s=up_1, up_2, \dots, up_m} = 0. \quad (3)$$

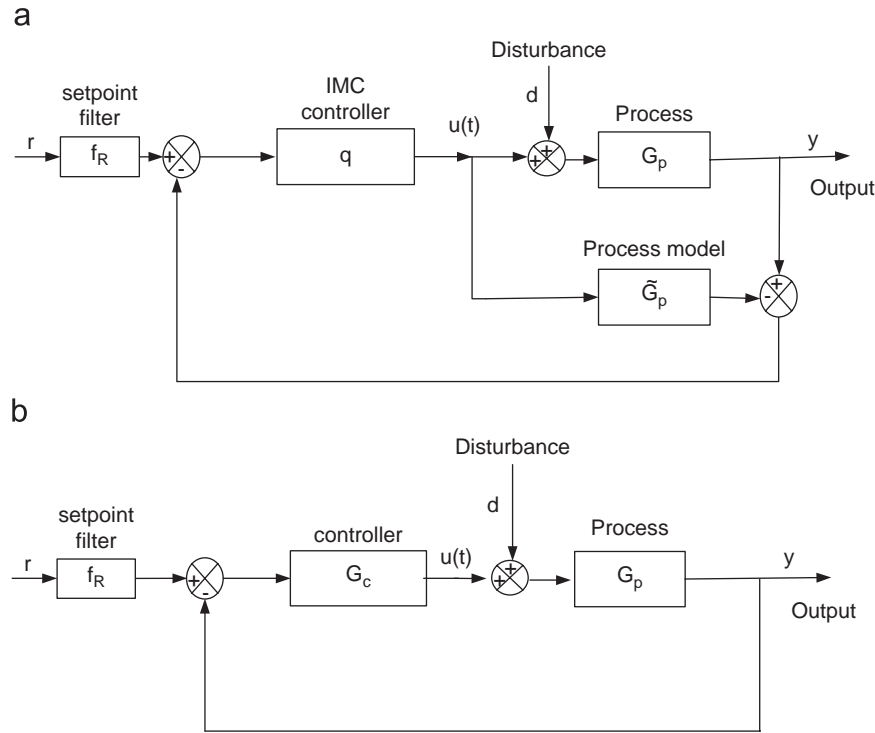


Fig. 1. Block diagram of IMC and classical feedback control: (a) the IMC structure and (b) classical feed back control structure.

Then, the IMC controller comes to be

$$q = p_m^{-1} \frac{(\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r} \quad (4)$$

Thus, the resulting setpoint and disturbance rejection is obtained as

$$\frac{y}{r} = G_p q f_R = \frac{p_A (\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r} f_R, \quad (5)$$

$$\frac{y}{d} = (1 - G_p q) G_p = \left( 1 - \frac{p_A (\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r} \right) G_p. \quad (6)$$

The numerator expression  $(\sum_{i=1}^m \alpha_i s^i + 1)$  in Eq. (5) causes an unreasonable overshoot in the servo response, which can be eliminated by adding the setpoint filter  $f_R$  as

$$f_R = \frac{1}{(\sum_{i=1}^m \alpha_i s^i + 1)}. \quad (7)$$

From Fig. 1, a feedback controller  $G_c$  which is equivalent to the IMC controller  $q$  is represented by

$$G_c = \frac{q}{1 - \tilde{G}_p q}. \quad (8)$$

The resulting ideal feedback controller is obtained as

$$G_c = \frac{p_m^{-1} \frac{(\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r}}{1 - \frac{p_A (\sum_{i=1}^m \alpha_i s^i + 1)}{(\lambda s + 1)^r}}. \quad (9)$$

Although the resulting controller in Eq. (9) does not have a PID controller structure, we can design a PID controller cascaded with a first order filter that resembles the equivalent feedback controller very closely. This will be discussed in the next section.

### 3. Proposed tuning rule

The first order delay unstable process (FODUP) is the representative model, which is commonly utilized for many unstable processes in the chemical process industry. Consequently, this section describes the design of the tuning rule for the FODUP and extends it to the DIP.

#### 3.1. First-order delay unstable process (FODUP)

$$G_p = \frac{K e^{-\theta s}}{\tau s - 1}, \quad (10)$$

where  $K$  is the gain,  $\tau$  the time constant, and  $\theta$  the time delay. The IMC filter structure exploited here is given as

$$f = \frac{\alpha s + 1}{(\lambda s + 1)^3}. \quad (11)$$

The resulting IMC controller can be obtained as follows:

$$q = \frac{(\tau s - 1)(\alpha s + 1)}{K (\lambda s + 1)^3}. \quad (12)$$

The IMC controller in Eq. (12) is proper and the ideal feedback controller which is equivalent to the IMC controller is

$$G_c = \frac{(\tau s - 1)(\alpha s + 1)}{K [(\lambda s + 1)^3 - e^{-\theta s} (\alpha s + 1)]}. \quad (13)$$

Approximating the dead time  $e^{-\theta s}$  with a  $\frac{1}{2}$  Pade expansion gives

$$e^{-\theta s} = \frac{(6 - 2\theta s)}{(6 + 4\theta s + \theta^2 s^2)}. \quad (14)$$

Substituting Eq. (14) for the dead time in Eq. (13) results in

$$G_c = \frac{(6+4\theta s+\theta^2 s^2)(\tau s-1)(\alpha s+1)}{K[(\lambda s+1)^3(6+4\theta s+\theta^2 s^2)-(\alpha s+1)(6-2\theta s)]}. \quad (15)$$

It is important to note that the  $1/2$  Pade approximation is precise enough to convert the ideal feedback controller into a PID cascaded first order filter with barely any loss of accuracy, while retaining the desired controller structure. Simplifying and rearranging Eq. (15), we obtain

$$G_c = \frac{(6 + 4\theta s + \theta^2 s^2)}{-K(6\theta + 18\lambda - 6\alpha)s} \times \frac{(-\tau s + 1)(\alpha s + 1)}{\left[1 + \frac{(2\alpha\theta + \theta^2 + 12\lambda\theta + 18\lambda^2)}{(6\theta + 18\lambda - 6\alpha)}s + \frac{(3\lambda\theta^2 + 12\lambda^2\theta + 6\lambda^3)}{(6\theta + 18\lambda - 6\alpha)}s^2 + \frac{(3\lambda^2\theta^2 + 4\lambda^3\theta)}{(6\theta + 18\lambda + 6\alpha)}s^3 + \frac{\lambda^3\theta^2}{(6\theta + 18\lambda - 6\alpha)}s^4\right]}. \quad (16)$$

It can be recognized from Eq. (16) that the resulting controller has the form of the PID controller cascaded with a high order filter. The analytical PID tuning formula can be obtained from Eq. (16) as

$$K_C = -\frac{4\theta}{K(6\theta + 18\lambda - 6\alpha)}, \quad (17a)$$

$$\tau_I = 2\theta/3, \quad (17b)$$

$$\tau_D = \theta/4. \quad (17c)$$

It is obvious from Eq. (3) that the denominator in Eq. (16) contains the factor  $(\tau s - 1)$ . Therefore, the filter parameter  $b$  in Eq. (1) can be obtained by taking the first derivative of Eq. (18) as

$$(ds^3 + cs^2 + bs + 1) = \frac{1 + \frac{(2\alpha\theta + \theta^2 + 12\lambda\theta + 18\lambda^2)}{(6\theta + 18\lambda - 6\alpha)}s + \frac{(3\lambda\theta^2 + 12\lambda^2\theta + 6\lambda^3)}{(6\theta + 18\lambda - 6\alpha)}s^2 + \frac{(3\lambda^2\theta^2 + 4\lambda^3\theta)}{(6\theta + 18\lambda - 6\alpha)}s^3 + \frac{\lambda^3\theta^2}{(6\theta + 18\lambda - 6\alpha)}s^4}{(-\tau s + 1)} \quad (18)$$

and substituting  $s = 0$  gives

$$b = \frac{(2\alpha\theta + \theta^2 + 12\lambda\theta + 18\lambda^2)}{(6\theta + 18\lambda - 6\alpha)} + \tau. \quad (19)$$

Since the high order  $(ds^3 + cs^2)$  term has little impact on the overall control performance in the control relevant frequency range, the remaining part of the fraction in Eq. (16) can be successfully approximated to a simple first order lead/lag filter  $(1 + as)/(1 + bs)$  where  $a = \alpha$ .

The value of  $\alpha$  is designed to remove the open-loop unstable pole at  $s = 1/\tau$ . This method chooses  $\alpha$  such that the term

$[1 - G_p q]$  has a zero at the pole of  $G_p$ , i.e.,  $[1 - G_p q]|_{s=1/\tau} = 0$ . Therefore, the designed value of  $\alpha$  is obtained from

$$\alpha = \tau \left[ \left(1 + \frac{\lambda}{\tau}\right)^3 e^{\theta/\tau} - 1 \right]. \quad (20)$$

### 3.2. Delay integrating process (DIP)

$$G_p = \frac{K e^{-\theta s}}{s}. \quad (21)$$

The DIP can be modeled by considering the integrator as an unstable pole near zero. This is mandatory since it is not practicable to implement the aforementioned IMC design procedure for the DIP, because the term,  $\alpha$ , vanishes at  $s = 0$ . As a result, the DIP can be approximated to the FODUP as follows:

$$G_p = \frac{K e^{-\theta s}}{s} = \frac{K e^{-\theta s}}{s - 1/\psi} = \frac{\psi K e^{-\theta s}}{\psi s - 1}, \quad (22)$$

where  $\psi$  is an arbitrary constant with a sufficiently large value. Accordingly, the optimum IMC filter structure for the DIP is identical to that for the FODUP model, i.e.,

$$f = (\alpha s + 1)/(\lambda s + 1)^3.$$

Therefore, the resulting IMC controller becomes  $q = (\psi s - 1)(\alpha s + 1)/K\psi(\lambda s + 1)^3$  and the subsequent PID tuning rules are obtained as

$$K_C = -\frac{4\theta}{K\psi(6\theta + 18\lambda - 6\alpha)}, \quad (23a)$$

$$\tau_I = 2\theta/3, \quad (23b)$$

$$\tau_D = \theta/4, \quad (23c)$$

$$a = \alpha, \quad (23d)$$

$$b = \frac{(2\alpha\theta + \theta^2 + 12\lambda\theta + 18\lambda^2)}{(6\theta + 18\lambda - 6\alpha)} + \psi, \quad (23e)$$

$$\alpha = \psi \left[ \left(1 + \frac{\lambda}{\psi}\right)^3 e^{\theta/\psi} - 1 \right]. \quad (23f)$$

**4. Simulation results**

This section is devoted to the simulation study which is classified as follows:

1. *Example 1.* Deals with a lag time dominant ( $\theta/\tau = 0.4$ ) FODUP. This is the most popular process model and has been included in performance comparisons by many researchers.
2. *Example 2.* Shows the performance superiority of the proposed method in the dead time dominant ( $\theta/\tau=1.2$ ) FODUP.
3. *Example 3.* Performance comparison for the DIP.
4. *Example 4.* Application of the proposed method to the distillation column model, which is widely used in the literature.

5. *Example 5.* Performance comparison of the proposed PID-filter controller with the modified SP controller.

In the simulation study, the performance and robustness of the control system were evaluated using the following indices to ensure a fair comparison.

4.1. Performance and robustness measure

4.1.1. Integral error criteria

To evaluate the closed-loop performance, the integral of the time-weighted absolute error (ITAE) criterion was considered in the case of both a step setpoint change and a step load

Table 1  
PID controller setting and performance matrix for Example 1

| Tuning methods          | $\lambda$ | $K_c$  | $\tau_I$ | $\tau_D$ | Ms   | Setpoint |           |      | Disturbance |           |       |
|-------------------------|-----------|--------|----------|----------|------|----------|-----------|------|-------------|-----------|-------|
|                         |           |        |          |          |      | ITAE     | Overshoot | TV   | ITAE        | Overshoot | TV    |
| Proposed <sup>a</sup>   | 0.2       | 0.4615 | 0.2667   | 0.10     | 3.04 | 0.616    | 1.01      | 2.94 | 0.757       | 0.613     | 3.785 |
| Liu et al. <sup>b</sup> | 0.5       | 2.634  | 2.5197   | 0.1541   | 3.04 | 0.405    | 1.0       | 3.50 | 1.516       | 0.694     | 3.586 |
| Lee et al. <sup>c</sup> | 0.5       | 2.634  | 2.5197   | 0.1541   | 3.04 | 1.276    | 1.0       | 1.44 | 1.516       | 0.694     | 3.586 |

<sup>a</sup> $a = 1.5779, b = 0.1053; f_R = 1/(1.5779s + 1).$

<sup>b</sup> $K_c = 2, C(s) = (s + 1)/(0.4s + 1).$

<sup>c</sup> $f_R = 1/(2.3566s + 1).$

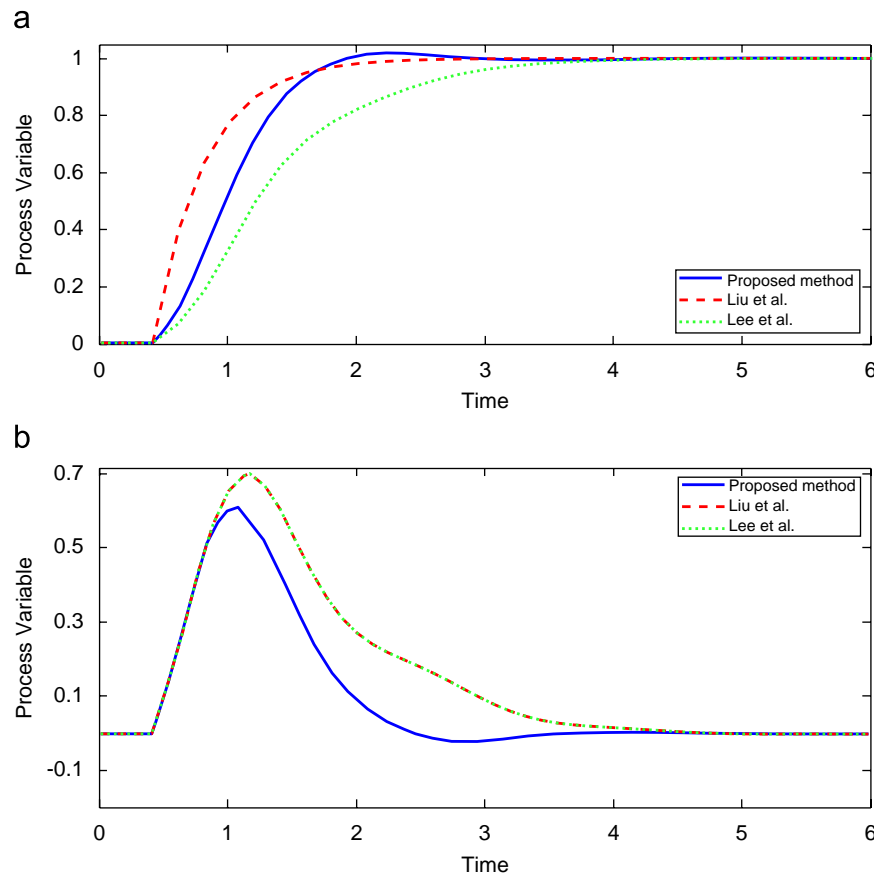


Fig. 2. Response of the nominal system for Example 1.

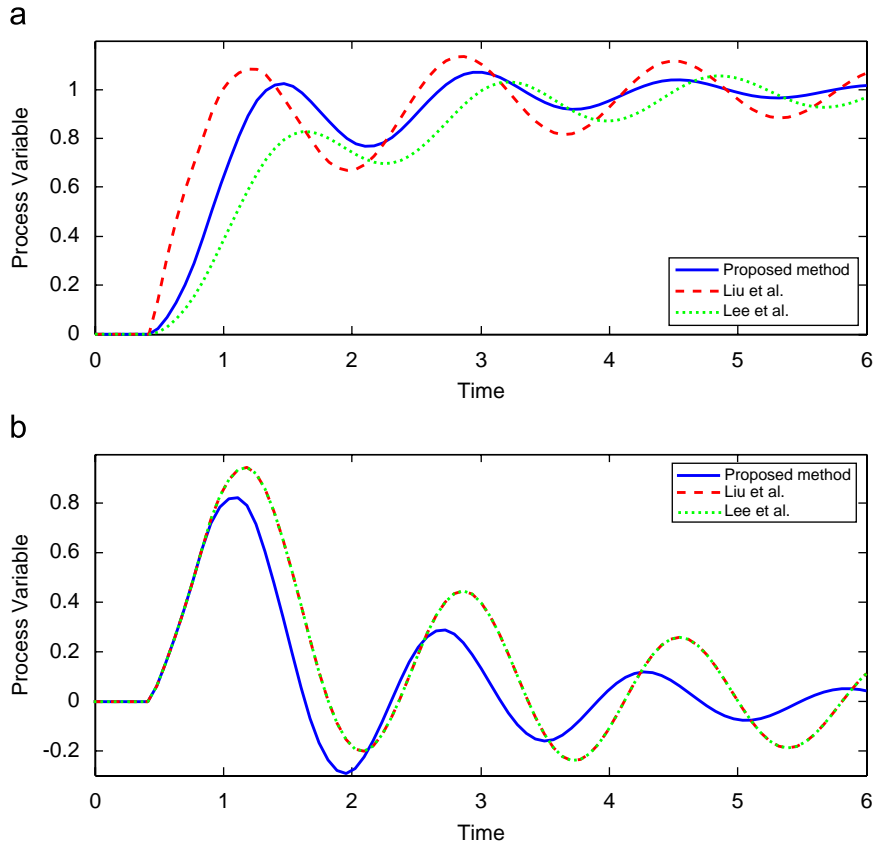


Fig. 3. Responses of the model mismatch system for Example 1.

disturbance. The ITAE is defined as

$$\text{ITAE} = \int_0^{\infty} t|e(t)| dt. \quad (24)$$

#### 4.1.2. Overshoot

Overshoot is a measure of how much the response exceeds the ultimate value following a step change in the setpoint and/or disturbance.

#### 4.1.3. Maximum sensitivity (Ms) to modeling error

To evaluate the robustness of a control system, the maximum sensitivity, Ms, which is defined as  $Ms = \max |1/[1 + G_p G_c(i\omega)]|$ , was used. Since Ms is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point  $(-1, 0)$ , a small Ms value indicates that the stability margin of the control system is large. Ms is a well-known robustness measure and has been used by many researchers (Chen and Seborg, 2002; Skogestad and Postlethwaite, 1996). To ensure a fair comparison, it is widely accepted for the model-based controllers (DS-d and IMC) to be tuned by adjusting  $\lambda$  so that the Ms values are the same. Therefore, throughout all of our simulation examples, all of the controllers compared were designed to have the same robustness level in terms of the maximum sensitivity, MS.

#### 4.1.4. Total variation (TV)

To evaluate the manipulated input usage, we compute the total variation (TV) of the input  $u(t)$  which is the sum of all its moves up and down. If we discretize the input signal as a sequence  $[u_1, u_2, u_3, \dots, u_i \dots]$ , then  $\text{TV} = \sum_{i=1}^{\infty} |u_{i+1} - u_i|$  should be as small as possible. The TV is a good measure of the smoothness of a signal (Skogestad and Postlethwaite, 1996).

#### 4.2. Example 1. Lag time dominant FODUP ( $\theta/\tau = 0.4$ )

An extensively published FODUP model (Huang and Chen, 1997; Lee et al., 2000; Tan et al., 2003; Liu et al., 2005; Majhi and Atherton, 2000) was considered for the performance comparison

$$G_p = \frac{1e^{-0.4s}}{1s - 1}. \quad (25)$$

For the above FODUP model, the recently published paper of Liu et al. (2005) demonstrated the superiority of their method over those of Tan et al. (2003) and Majhi and Atherton (2000). In this simulation study, the proposed method was compared with those of Liu et al. (2005) and Lee et al. (2000). The design of the disturbance rejection is identical for both Liu et al.'s (2005) and Lee et al.'s (2000) methods. However, for the setpoint response, Liu et al. (2005) used a modified IMC



Table 2  
PID controller setting and performance matrix for Example 2

| Tuning methods          | $\lambda$   | $K_c$  | $\tau_I$ | $\tau_D$ | Ms   | Setpoint |           |       | Disturbance |           |       |
|-------------------------|---|--------|----------|----------|------|----------|-----------|-------|-------------|-----------|-------|
|                         |   |        |          |          |      | ITAE     | Overshoot | TV    | ITAE        | Overshoot | TV    |
| Proposed <sup>a</sup>   | 1.1   | 0.0317 | 0.8      | 0.3      | 10.7 | 13.67    | 1.02      | 1.288 | 15.45       | 0.544     | 1.455 |
| Tan et al.              | $K_0 = 2, K_1 = (s + 1)/(2s + 1), K_2 = 1.1(0.49s + 1)$ |        |          |          |      | 7.12     | 1.0       | 1.50  | 17.97       | 0.667     | 1.588 |
| Lee et al. <sup>b</sup> | 2.936   | 1.1764 | 51.013   | 0.5729   | 10.7 | 34.65    | 1.0       | 1.067 | 33.21       | 0.615     | 1.668 |

<sup>a</sup> $a = 29.7476, b = 0.2709; f_R = 1/(29.7476s + 1).$

<sup>b</sup> $f_R = 1/(50.4356s + 1).$

structure, while Lee et al. (2000) applied a setpoint filter. For the methods of both Liu et al. (2005) and Lee et al. (2000),  $\lambda = 0.5$  was used in the simulation, which results in  $Ms = 3.03$ . To obtain a fair comparison,  $\lambda$  was also adjusted in the proposed method ( $\lambda = 0.20$ ) to obtain  $Ms = 3.03$ . The controller parameters, including the performance and robustness matrix, are listed in Table 1.

Fig. 2 shows the comparison of the proposed method with those of Liu et al. (2005) and Lee et al. (2000), performed by introducing a unit step change in both the setpoint and load disturbance. For the servo response, the setpoint filter is used for both the proposed method and that of Lee et al. (2000), whereas a three controller element structure is used for the method of Liu et al. (2005). As is apparent from Fig. 2 and Table 1, the proposed method results in an improved load disturbance response. Since the design of the disturbance rejection is identical for both Liu et al.'s (2005) and Lee et al.'s (2000) methods, the same PID tuning setting and consequently an identical disturbance rejection response is obtained in both cases. For the servo response, the method of Liu et al. (2005) seems better, but the settling times of Liu et al.'s (2005) method and the proposed method are comparable, while Lee et al.'s method (2000) shows the slowest response with a long settling time.

It is important to note that the well-known modified IMC structure has the theoretical advantage of eliminating the time delay from the characteristic equation. Unfortunately, this advantage is lost if the process model is inaccurate. Besides, real process plants usually incorporate unmodeled dynamics that inevitably tend to deteriorate the control system performance severely. The robustness of the controller was investigated by inserting a perturbation uncertainty of 10% in all three parameters simultaneously toward the worst case model mismatch, i.e.,  $G_p = 1.1e^{-0.44s}/(0.9s - 1)$ . The simulation results for the model mismatch case are presented in Fig. 3 for both the setpoint tracking and the disturbance rejection. It is obvious from Fig. 3 that the proposed controller tuning method has an excellent setpoint and load response, while the modified IMC controller corresponding to Liu et al.'s (2005) method has the worst setpoint response for the model mismatch. The better setpoint response for the nominal case afforded by the SP controller is achieved by sacrificing the robustness of the closed-loop system. For the disturbance rejection, the methods of Liu et al. (2005) and Lee et al. (2000) are identical and perfectly overlapped.

#### 4.3. Example 2. Dead time dominant FODUP ( $\theta/\tau = 1.2$ )

Consider an unstable dead time dominant process (Huang and Chen, 1997; Tan et al., 2003) as follows:

$$G_p = \frac{1e^{-1.2s}}{(1s - 1)}. \quad (26)$$

Tan et al. (2003) previously demonstrated the superiority of their method over other methods including that of Huang and Chen (1997). The  $\lambda$  value for both the proposed method and Lee et al.'s (2000) method was adjusted for  $Ms = 10.71$ . The controller setting parameters are listed in Table 2. To test the performance of the control system, the load disturbance has a step change of magnitude 0.1, and a setpoint with a magnitude of 1 is added. The simulation results are provided in Fig. 4 for both the setpoint tracking and the disturbance rejection. It is clear from Fig. 4 and Table 2 that the proposed method results in an improved load disturbance response. The proposed method shows superiority for the load disturbance over the other controllers. The setpoint response given by the method of Tan et al. (2003) is the best among all of the methods, whereas Lee et al.'s (2000) method has the slowest response with a long time to reach the steady state. It is important to note that Tan et al.'s (2003) method has a modified IMC structure using three individual controllers. In the proposed method, the servo response is initially slow, but the settling times for both the proposed method and Tan et al.'s (2003) method are similar. The modified IMC structure proposed by Tan et al. (2003) has the merit of providing a nominal setpoint response, but it loses when the process has unmodeled dynamics.

The robustness of the controller is evaluated by inserting a perturbation uncertainty of 5% in all three parameters simultaneously to obtain the worst case model mismatch, i.e.,  $G_p = 1.05e^{-1.26s}/(0.95s - 1)$  as an actual process, whereas the controller settings are those calculated for the process with the nominal model. Fig. 5 shows both the setpoint and disturbance rejection responses for model mismatch. The controller settings of the proposed method provide the most robust performance for both the servo and regulatory problems. The methods of Tan et al. (2003) and Lee et al. (2000) give an unstable oscillatory response for both the setpoint and disturbance rejection, as is apparent from Fig. 5.

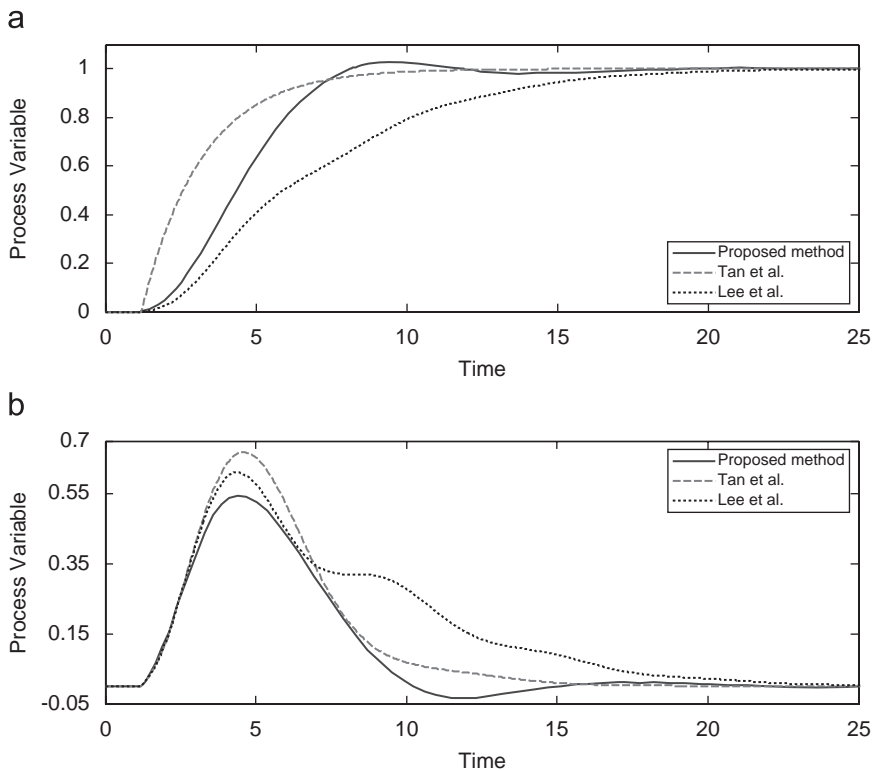


Fig. 4. Response of the nominal system for Example 2.

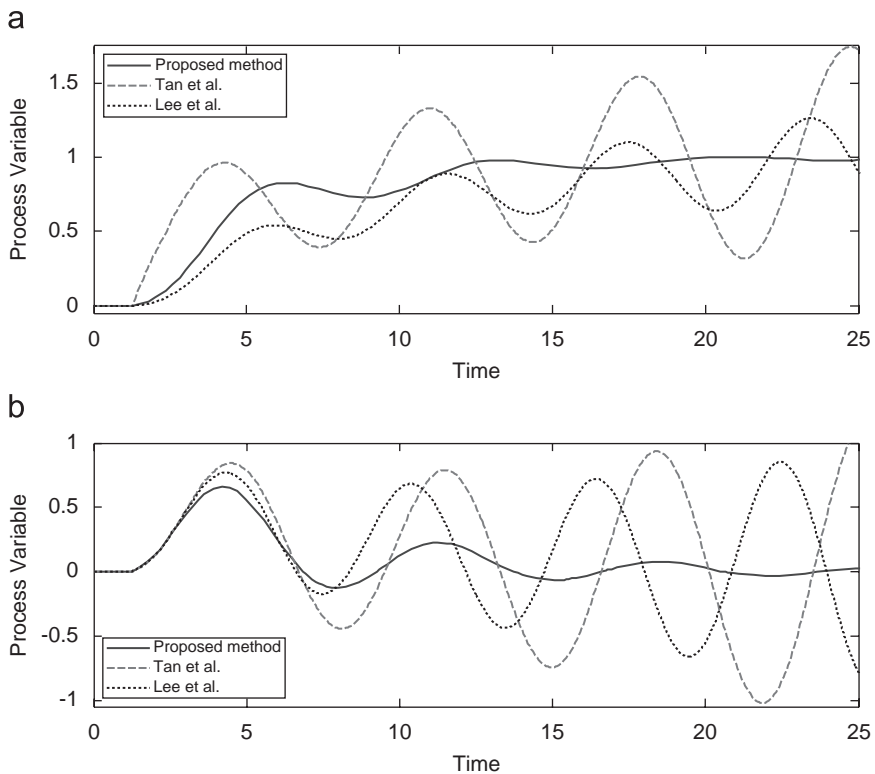


Fig. 5. Responses of the model mismatch system for Example 2.



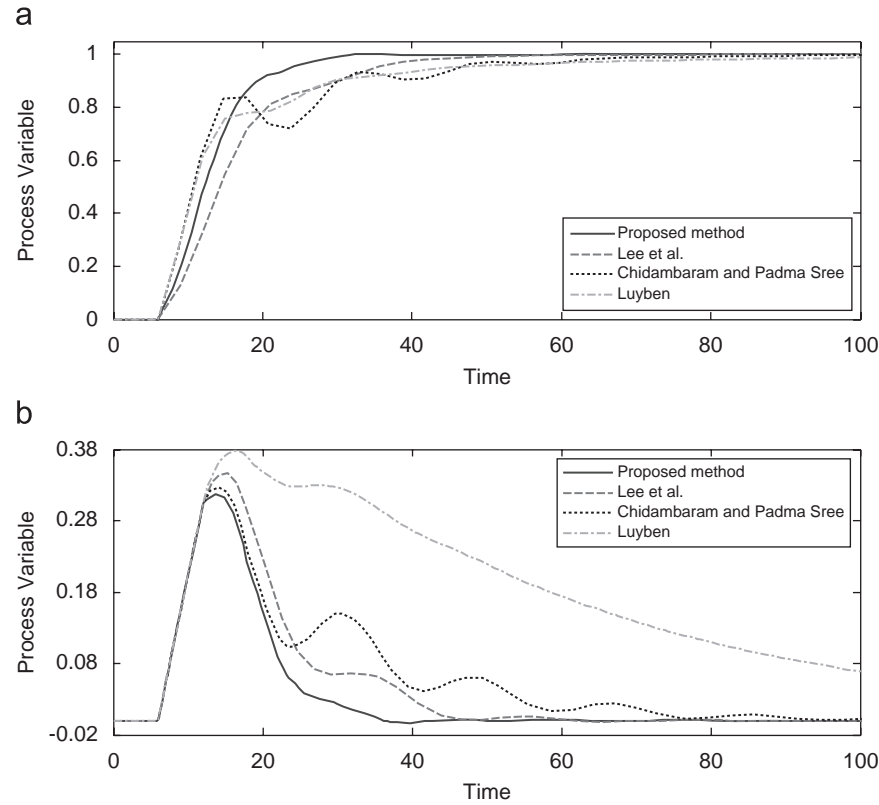


Fig. 6. Response of the nominal system for Example 3.

Table 3  
PID controller setting and performance matrix for Example 3

| Tuning methods                          | $\lambda$ | $K_c$ | $\tau_I$ | $\tau_D$ | Ms   | Setpoint |           |      | Disturbance |           |      |
|---|-----------|-------|----------|----------|------|----------|-----------|------|-------------|-----------|------|
|   |           |       |          |          |      | ITAE     | Overshoot | TV   | ITAE        | Overshoot | TV   |
| Proposed <sup>a</sup>                   | 2.155     | 1.079 | 4.0      | 1.5      | 2.79 | 101.4    | 1.0       | 4.59 | 60.2        | 0.317     | 4.65 |
| Lee et al. <sup>b</sup>                 | 4.838     | 3.686 | 19.22    | 2.24     | 2.79 | 179.5    | 1.0       | 3.14 | 102.1       | 0.346     | 3.00 |
| Chidambaram and Padma Sree <sup>c</sup> | –         | 4.066 | 27.0     | 2.7      | 3.81 | 239.0    | 1.0       | 8.19 | 178.9       | 0.327     | 5.04 |
| Luyben <sup>d</sup>                     | –         | 2.563 | 56.32    | 3.561    | 2.24 | 302.2    | 1.0       | 3.95 | 1053        | 0.377     | 1.93 |

<sup>a</sup> $a = 13.1974$ ,  $b = 1.4414$ ;  $f_R = 1/(13.1974s + 1)$ .

<sup>b</sup> $f_R = 1/(16.7065s + 1)$ .

<sup>c</sup> $f_R = (10.80s + 1)/(72.9s^2 + 27s + 1)$ .

<sup>d</sup> $G_c = K_c(1 + 1/\tau_I s + \tau_D s)/(1 + b_L s)$  where  $b_L = 0.382$ ;  $f_R = (39.42s + 1)/(200.55s^2 + 56.32s + 1)$ .

#### 4.4. Example 3: DIP process

The following DIP model was considered which was previously studied by other researchers (Luyben, 1996; Visioli, 2001; Chidambaram and Padma Sree, 2003):

$$G_p = \frac{0.0506e^{-6s}}{s} \quad (27)$$

Chidambaram and Padma Sree previously demonstrated the superiority of their method over that of Visioli (2001). In the simulation study, we compared the proposed method, with those of Lee et al. (2000), Luyben (1996), and Chidambaram and Padma Sree (2003). For both the proposed method and that of Lee et al. (2000),  $\lambda$  was adjusted to obtain  $Ms = 2.79$ . The

design method of Chidambaram and Padma Sree's (2003) and Luyben's (1996) methods is not based on the  $\lambda$  tuning method and we therefore used their respective values without adjusting the  $Ms$  value.

The proposed controller was designed by considering the DIP as  $G_p = 5.06e^{-6s}(100s - 1)$ . Fig. 6 shows the closed-loop output response for a unit step change in both the setpoint and load disturbance. The controller setting parameters and performance matrix for both the setpoint and load disturbance are listed in Table 3. To eliminate the overshoot in the setpoint response, a setpoint filter is used in the proposed method and that of Lee et al. (2000), while in the methods of Chidambaram and Padma Sree (2003) and Luyben (1996), a setpoint weighting type filter.

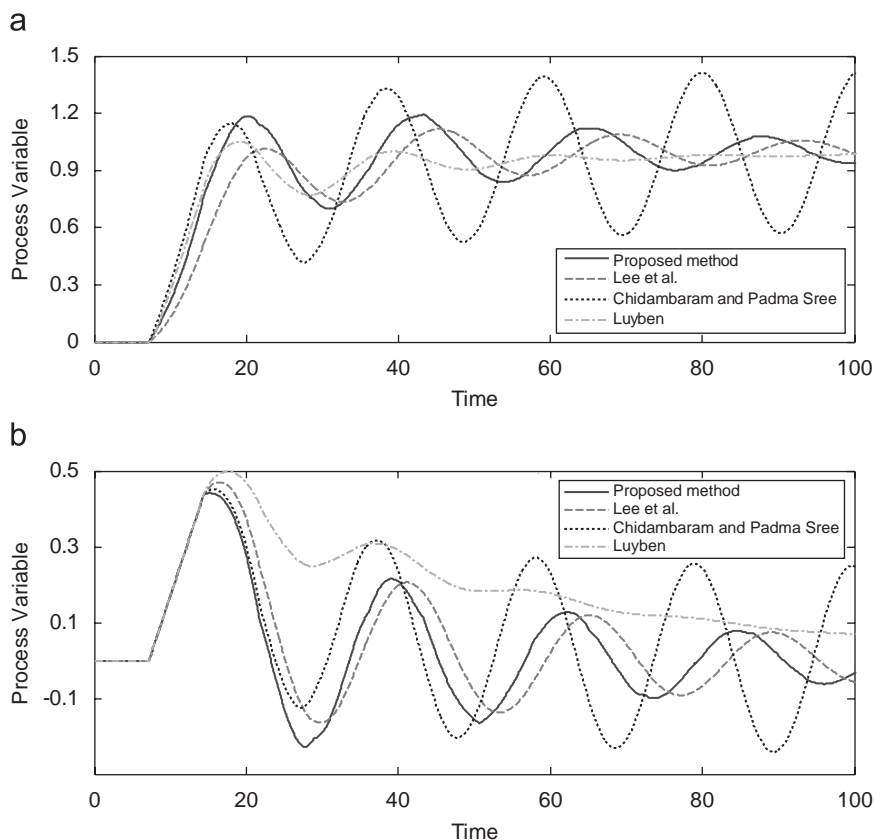


Fig. 7. Responses of the model mismatch system for Example 3.

$f_R = (\varepsilon\tau_I S + 1)/(\tau_I\tau_D S^2 + \tau_I S + 1)$ , where  $0 \leq \varepsilon \leq 1$ , is used. In Chidambaram and Padma Sree's paper, the recommended value of  $\varepsilon = 0.4$ , while in the Luyben method (1996)  $\varepsilon = 0.7$  is employed. On the basis of the comparison of the output response and the values of the performance matrices listed in Table 3, it is apparent that the proposed controller shows the best performance. To confirm the robust performance of the proposed method, it was assumed that there are 20% parameter perturbations in  $K$  and  $\theta$  simultaneously toward the worst case model mismatch, i.e.,  $G_p = 0.0607e^{-7.2s}/s$ . As shown in Fig. 7, both the proposed method and that of Luyben (2003) are robust to parameter perturbation. Note that the robust performance of Luyben's (2003) method is achieved at the expense of the sluggish nominal response. The proposed method has better performance indices in the case of both the nominal and model mismatches when it is tuned to have the same  $M_s$  as Luyben's method (2003).

#### 4.5. Example 4. Distillation column model

Distillation remains the most widely used separation technique in the petrochemical and chemical process industries for the separation of fluid mixtures. The operation of the distillation column is extremely critical, because of the purity requirements of the products. The distillation column separates a small amount of a low-boiling material from the final product. The bottom level of the distillation column is controlled by

adjusting the steam flow rate. The process model for the level control system is represented by the DIP. The distillation column model studied by Chien and Fruehauf (1990) and Chen and Seborg (2002) was considered for the present study as follows:

$$G_p = \frac{0.2e^{-7.4s}}{s}. \quad (28)$$

The methods proposed by Chen and Seborg (2002) and Lee et al. (2000) were used to design the PID controller, as shown in Fig. 8. The  $\lambda$  value was selected for each method to give  $M_s = 1.90$ . The controller settings are listed in Table 4. The proposed controller was designed by considering the DIP as  $G_p = 20e^{-7.4s}/(100s - 1)$ .

Fig. 8 shows the output response, where the proposed tuning rule results in the least settling time for both the servo and disturbance rejection, followed by that of Chen and Seborg (2002). Lee et al.'s (2000) method has the slowest response and requires the longest settling time for both the setpoint and disturbance rejection. A setpoint weighting type filter is used for the method of Chen and Seborg (2002) to reduce the overshoot in the setpoint response. On the basis of Fig. 8 and the performance indices listed in Table 4, it is evident that the proposed method performs better than the other conventional methods for both the servo and regulatory problems.

The robustness of the controller is also evaluated by inserting a perturbation uncertainty of 75% in the gain and 20% in

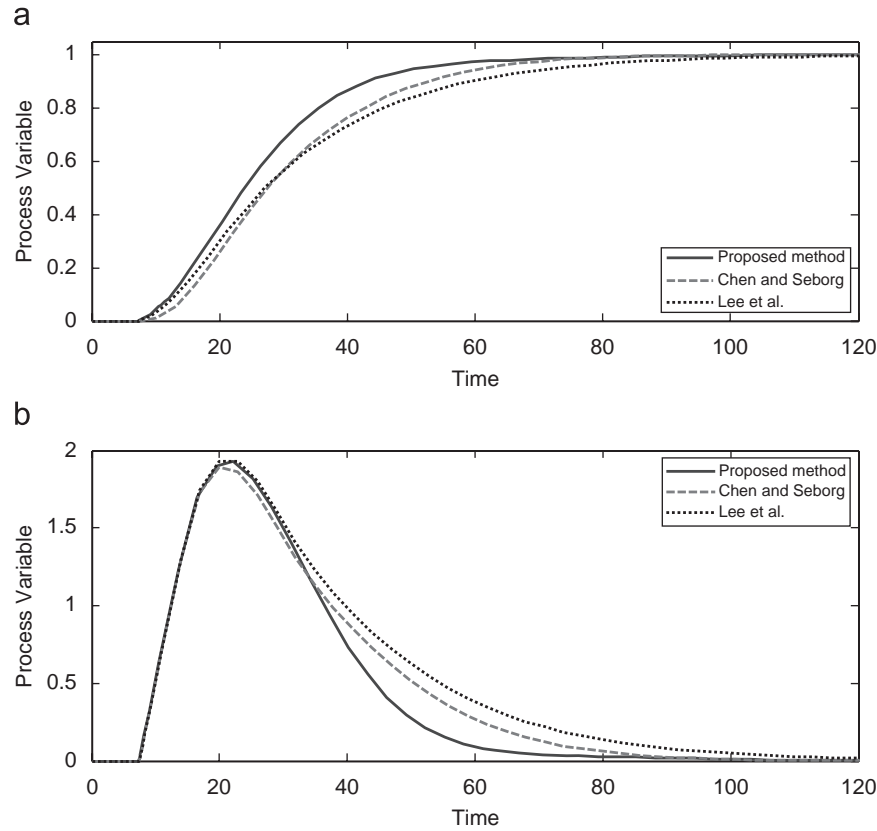


Fig. 8. Response of the nominal system for Example 4.

Table 4  
PID controller setting and performance matrix for Example 4

| Tuning methods               | $\lambda$ | $K_c$  | $\tau_I$ | $\tau_D$ | Ms   | Setpoint |           |       | Disturbance |           |       |
|------------------------------|-----------|--------|----------|----------|------|----------|-----------|-------|-------------|-----------|-------|
|                              |           |        |          |          |      | ITAE     | Overshoot | TV    | ITAE        | Overshoot | TV    |
| Proposed <sup>a</sup>        | 5.56      | 0.0956 | 4.9333   | 1.85     | 1.90 | 452.7    | 1.0       | 0.381 | 1470.0      | 1.92      | 1.814 |
| Chen and Seborg <sup>b</sup> | 9.15      | 0.5432 | 31.15    | 2.558    | 1.90 | 604.9    | 1.0       | 0.318 | 1794.0      | 1.93      | 1.845 |
| Lee et al. <sup>c</sup>      | 11.0      | 0.5367 | 35.137   | 2.286    | 1.90 | 737.0    | 1.0       | 0.303 | 2292.0      | 1.98      | 1.786 |

<sup>a</sup>  $a = 26.659, b = 3.0952; f_R = 1/(26.659s + 1)$ .

<sup>b</sup>  $f_R = 1/(79.691s^2 + 31.15s + 1)$ .

<sup>c</sup>  $f_R = 1/(32.6734s + 1)$ .

the dead time simultaneously toward the worst case model mismatch, as follows:

$$G_p = 0.35e^{-8.88s}/s.$$

The simulation results for the plant-model mismatch are given in Fig. 9 for the both servo and regulatory problems. It should be mentioned that the controller settings used in simulation are those calculated for the process with nominal process parameters. The responses indicate that the proposed method has less oscillatory response as well as the minimum settling time for both the setpoint and disturbance rejection. The method of Chen and Seborg (2002) shows more oscillation, followed by that of Lee et al. (2000). It seems that the proposed method gives good performance, even for severe process uncertainties.

#### 4.6. Example 5. Comparison with the modified SP

The proposed controller is compared with the modified SP (Zhang et al., 2004) given in Fig. 10, which has a more complicated structure with three controllers. For the purpose of comparison, we consider the process described below which was studied by Zhang et al. (2004), Majhi and Atherton (2000), and Kwak et al. (1999):

$$G_p = \frac{1e^{-0.5s}}{1s - 1}. \tag{29}$$

The proposed method was compared with that of Zhang et al. (2004) because their methods were shown to be superior to the previously reported SP methods, such as those of

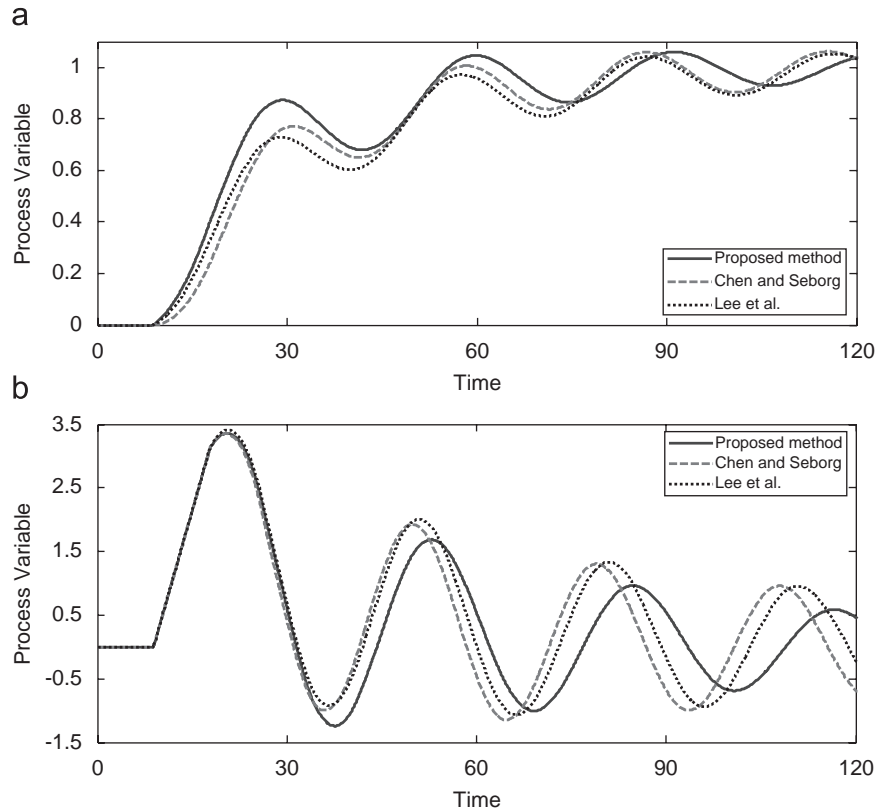


Fig. 9. Responses of the model mismatch system for Example 4.

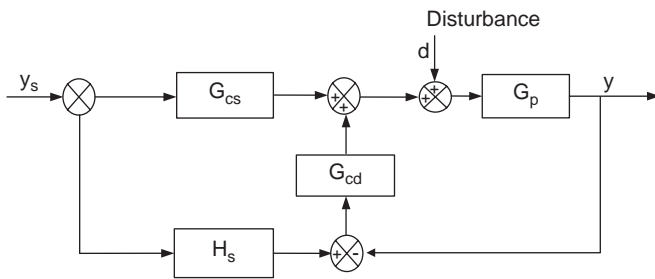


Fig. 10. Simplified structure for the modified Smith predictor.

Majhi and Atherton (2000) and Kwak et al. (1999). The controllers given by Zhang et al. (2004) are

$$C_{cs}(s) = \frac{s-1}{0.5s+1}, \quad (30a)$$

$$H_s(s) = \frac{e^{-0.5s}}{0.5s+1}, \quad (30b)$$

$$C_{cd}(s) = 2.6483 \left( 1 + \frac{1}{2.4669} + 0.2185s \right). \quad (30c)$$

In the proposed controller,  $\lambda = 0.235$  is selected and the resulting tuning parameters are obtained as  $K_c = 0.3701$ ,  $\tau_I = 0.3333$ ,  $\tau_D = 0.125$ ,  $a = 2.1056$  and  $b = 0.1192$ . The simulation was conducted by inserting a unit step change in both

the setpoint and load disturbance. For the servo response, the setpoint filter  $f_R = 1/(2.1056s + 1)$  is used for the proposed method.

Fig. 11 shows a comparison of the nominal response obtained by the proposed PID-filter controller and that obtained by Zhang et al.'s (2004) modified SP controller. Note that the proposed controller uses a simple feedback control structure without any dead time compensator. Nevertheless, the proposed controller provides superior performance, as shown in Fig. 11. The disturbance rejection afforded by the proposed controller has a smaller settling time, whereas the modified SP controller described by Zhang et al. (2004) shows an aggressive response with significant overshoot and oscillation that requires a long time to settle.

As regards the servo response, the modified SP controller has an initially fast response, because of the elimination of the dead time. The proposed method has an initially slow response, but the settling time is similar to that afforded by the modified SP controller.

It is important to note that the SP control configuration has the clear advantage of eliminating the time delay from the characteristic equation, which is very effective in improving the setpoint tracking performance. However, this advantage is lost if the process model is inaccurate. In order to evaluate the robustness against model uncertainty, a simulation study was conducted for the worst case model mismatch by assuming that the process has a 5% mismatch in the three process parameters in

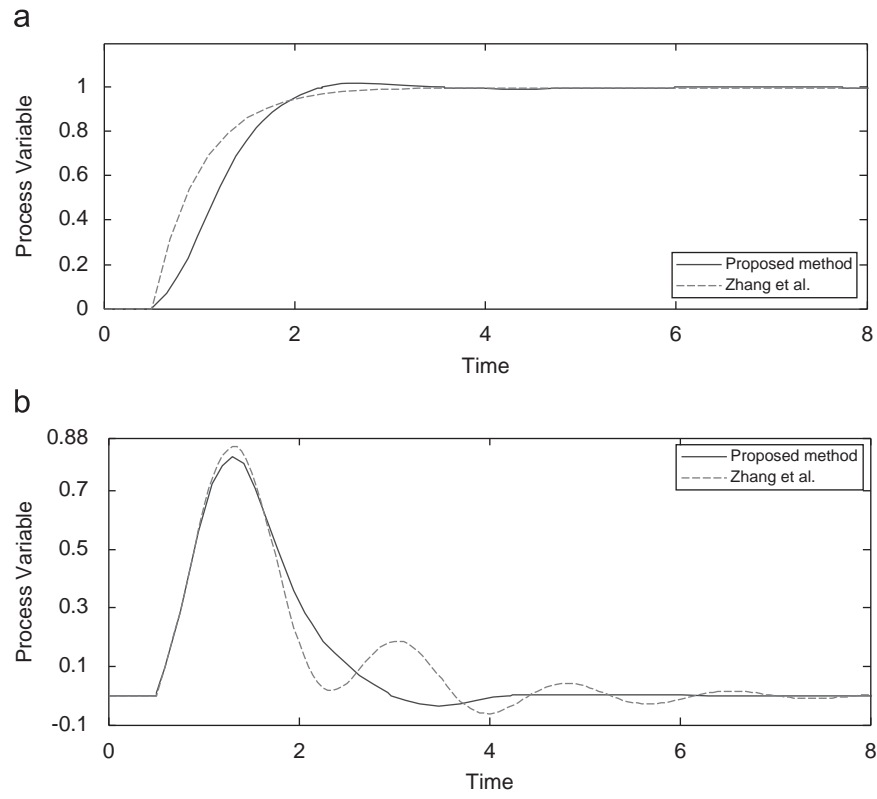


Fig. 11. Response of the nominal system for Example 5.

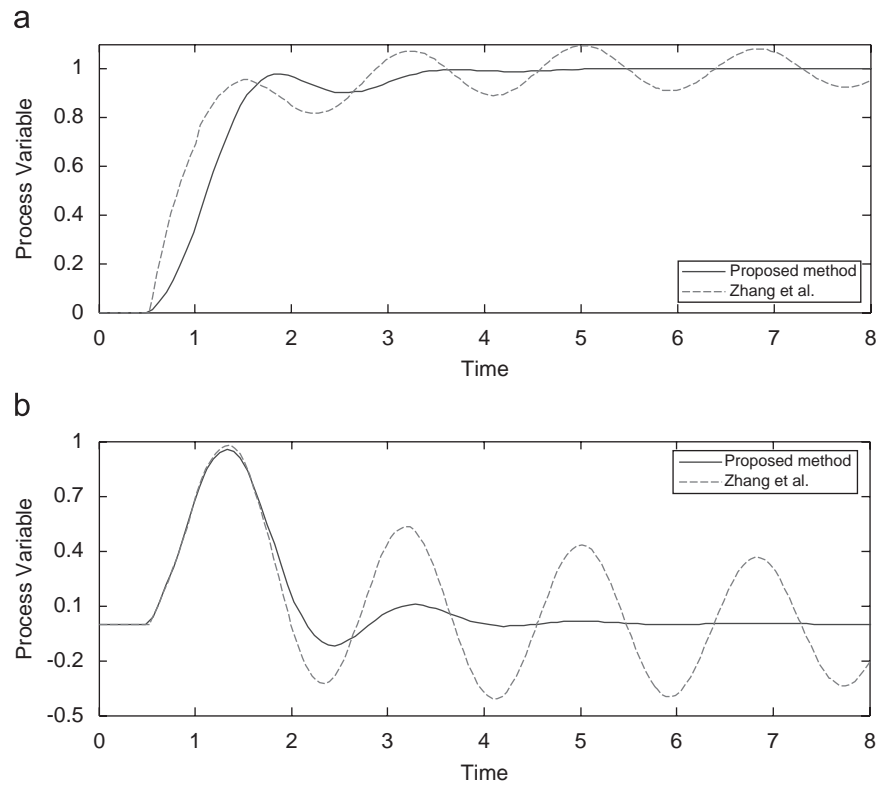


Fig. 12. Responses of the model mismatch system for Example 5.

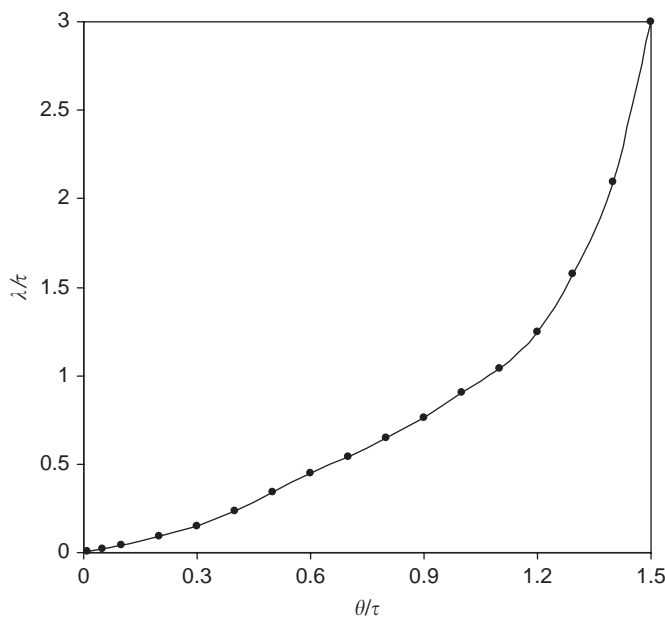


Fig. 13.  $\lambda$  guideline for FODUP.

the worst direction, as follows:

$$G_p = \frac{1.05e^{-0.525s}}{0.95s - 1} \quad (31)$$

The closed-loop responses are presented in Fig. 12. Notice that the modified SP method described by Zhang et al. (2004) gives a severe oscillatory response on the verge of instability for both the servo and regulatory problems, whereas the proposed controller gives a more robust response. In practice, the robustness is as important as the nominal performance. One key in designing the controller is the tradeoff between its robustness and nominal performance. As shown in Fig. 12, the proposed method provides not only better nominal performance but also excellent robustness, while using a simple feedback control structure.

#### 4.7. Closed-loop time constant $\lambda$ guideline

The closed-loop time constant  $\lambda$  is a user-defined tuning parameter in the proposed tuning rule. It is directly related to the performance and robustness of the proposed tuning method and, therefore, it is important to have a guideline for setting the  $\lambda$  value in order to provide both a fast and robust response for a given  $\theta/\tau$  ratio.

Fig. 13 shows the plot of  $\lambda/\tau$  versus  $\theta/\tau$  for the FODUP model. It is important to notice that the desirable  $M_s$  value to give robust control performance in an unstable process tends to gradually increase as the dead time increases. In Fig. 13, for instance, the  $M_s$  values corresponding to the recommended  $\lambda$  values are approximately  $M_s = 3.0$  for  $\theta/\tau \leq 0.5$ ;  $M_s = 4.0$  for  $0.5 \leq \theta/\tau \leq 0.6$ ; and  $M_s = 5.0$  for  $0.6 \leq \theta/\tau \leq 0.8$ . If the resulting closed-loop performance or robust stability is not acceptable, then the  $\lambda$  value should be monotonously increased or decreased

until the desirable trade-off between the nominal and robust performances is achieved.

## 5. Conclusions

A simple analytical design method for a PID-filter controller was proposed based on the IMC principle for the FODUP and DIP processes. The proposed PID-filter controller can easily be implemented on the modern control hardware. The proposed method affords an excellent improvement in both the setpoint and disturbance rejection for the FODUP and DIP processes. Several representative processes frequently used in many previous studies were considered in the simulation study. The simulation was conducted by tuning the various controllers to have the same degree of robustness in terms of  $M_s$  value in order to provide a fair comparison. The proposed controller consistently provided superior performance over the whole range of the  $\theta/\tau$  ratio. The robustness study was conducted by inserting a perturbation uncertainty in all parameters simultaneously to obtain the worst case model mismatch, and the proposed method was found to be superior to the other methods. The proposed controller was also compared with more sophisticated controllers such as the modified SP. The result showed that the proposed controller gives satisfactory performance in both the nominal and model mismatch cases, without any external dead time compensator. The closed-loop time constant,  $\lambda$ , guideline was also proposed for a wide range of  $\theta/\tau$  ratios.

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