

## PID controller design for integrating processes with time delay

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**Abstract**—A simple IMC-PID controller design technique is proposed on the basis of the IMC principle for two representative integrating processes with time delay. Further, it is extended to integrating processes with negative and positive zero as well. The proposed PID controller design method is mainly focused on the disturbance rejection, which causes the overshoot in the setpoint response, and a two-degree-of-freedom (2DOF) control structure is used to eliminate this overshoot. The simulation results show the superiority of the proposed tuning rule over other existing methods, when the controller is tuned to have the same robustness level by evaluating the peak of the maximum sensitivity ( $M_s$ ). The closed loop time constant ( $\lambda$ ) has only one user-defined tuning parameter in the proposed method. A guideline is suggested for the selection of  $\lambda$  for different robustness levels by evaluating the value of  $M_s$  over a wide range of  $\theta/\tau$  ratios.

Key words: PID Controller Tuning, Integrating Process with Time Delay, Disturbance Rejection, Two-Degree-of-Freedom Controller

### INTRODUCTION

Many units used in the chemical process industry, such as heating boilers, batch chemical reactors, liquid storage tanks or liquid level systems, are integrating processes in which the dynamic response is very slow with a large dominant time constant. Due to transportation delays in the recycle loops and composition analysis loops etc., a time delay exists in the majority of processes used in the process industries.

In process control, the majority of the control loops are of the proportional-integral-derivative (PID) type at the regulatory level. The main reason for this is their relatively simple structure, which can be readily understood and allows them to be easily implemented in practice. Finding design methods that lead to the optimal operation of the PID controllers is therefore of significant interest. Integrating processes or first order systems with an integrator (with/without zero) are frequently encountered in the process industries. For first order systems with an integrator and with/without zero, if the zero is positive, the system exhibits an inverse response; if the zero is negative, then the system shows large overshoot in the response.

Model-based control strategies such as the internal model control (IMC) and direct synthesis methods have been proposed by several authors [1-7] to enhance the closed loop performance of integrating processes with time delay.

Some of the most important methods for the design of first order processes with an integrator are those of Skogestad [1] and Zhang et al. [2], which are based on the IMC control design methodology, and Chen and Seborg [3] who used the direct synthesis approach, whereas Wang and Cai [8] used the gain and phase margin specifications to calculate the PID parameters.

Recently, the design of the double integrating process has become very popular, since it is widely used in industrial processes such as aerospace control systems, DC motors and high speed disk drives

whose dynamics show the characteristics of the double integrator type. The controller design methods for these types of processes have been addressed by Skogestad [1] and Liu et al. [6].

The classical example of an integrating process with an inverse response is the level control of a boiler steam drum. The “boiler swell” problem can lead to a transfer function between the drum level and boiler feed water flow rate that contains a pure integrator and a positive zero, in addition to some dead time and lags.

In practice, the constraints on the steam drum level are important. If the liquid level grows too high, liquid will enter the superheater section above the steam drum, expanding rapidly and causing the “riser” pipe to rupture. If the liquid level gets too low, there will be no more water in the “downcomer” pipes in the radiation section below the steam drum, causing the pipes to get too hot and break down. On the other hand, because of the combination of the integration and inverse response, the control of the steam drum level is tougher than most other level-control problems.

Gu et al. [7] developed an analytical design procedure for PI/PID controllers on the basis of  $H_\infty$  optimization and IMC theory. Earlier, Luyben [9] presented an identification method for this type of system from step-response data. On the basis of this, the PI and PID tuning methods were discussed in his paper.

In the open literature, controller design methods for first order integrating process with one negative zero were reported by Shamsuzzoha et al. [10] and Wang and Cluett [11] and are most useful to represent the control system of the paper drum dryer cans.

The delay integrating process has a clear advantage in the identification test, because the model contains only two parameters and is simple to use for identification. Some of the well accepted PID controller tuning methods for delay integrating processes are those proposed by Chien and Fruehauf [12], Luyben [13], Chen and Seborg [3] and Chidambaram and Sree [14].

Due to the simplicity and superior performance of the IMC-based tuning rule, the analytically derived IMC-PID tuning [15-18] methods have attracted the attention of industrial users recently. The IMC-PID tuning rule has only one user-defined tuning parameter, which

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is directly related to the closed-loop time constant. It is apparent from literature that the single input single output (SISO) PID controller design is valuable for real process plant. The multiloop PID controller is also commonly [19-21] used in the multiple input multiple output process.

Despite the fact that the PID controller design method for several kinds of integrating process has been discussed extensively in the literature, the design of a controller which is simple and robust with improved performance remains an open issue.

Therefore, the present work is devoted to the design of PID controllers for several classes of integrating processes in a unified framework. The proposed method is developed on the basis of the IMC principle for disturbance rejection. The controller design for disturbance rejection provides excessive overshoot in the servo response, so the concept of the 2DOF control structure is used to cope with the setpoint performance. The performance of the proposed tuning rule is compared with other existing methods, both in the nominal and model mismatch cases. A guideline is suggested for the selection of  $\lambda$  for different robustness levels by evaluating the  $M_s$  value over a wide range of  $\theta/\tau$  ratios. An IAE comparison is also performed at a fixed  $M_s$  for several tuning methods, which clearly indicates that the proposed method gives consistently better performance over a broad range of  $\theta/\tau$  ratios.

**CONTROLLER DESIGN ALGORITHM**

Fig. 1 shows the block diagram of the IMC control and equivalent classical feedback control structures, where  $G_p$  is the process,  $\tilde{G}_p$  is the process model, and  $Q$  is the IMC controller. The controlled variables are related as follows:

$$y = \frac{G_p Q f_r}{1 + Q(G_p - \tilde{G}_p)} r + \left[ \frac{1 - \tilde{G}_p Q}{1 + Q(G_p - \tilde{G}_p)} \right] G_p d \tag{1}$$

For the nominal case (i.e.,  $G_p = \tilde{G}_p$ ), the setpoint and disturbance re-

sponses are simplified as

$$\frac{y}{r} = G_p Q f_r \tag{2}$$

$$\frac{y}{d} = [1 - \tilde{G}_p Q] G_p \tag{3}$$

**1. IMC Controller Design Steps**

The IMC controller design involves two steps:

*Step 1:* The process model  $\tilde{G}_p$  is factored into invertible and non invertible parts

$$\tilde{G}_p = P_M P_A \tag{4}$$

where  $P_M$  is the portion of the model inverted by the controller;  $P_A$  is the portion of the model not inverted by the controller (it is usually a non-minimum phase and contains dead times and/or right half plane zeros);  $P_A(0) = 1$ .

*Step 2:* The idealized IMC controller is the inverse of the invertible portion of the process model.

$$\tilde{Q} = P_M^{-1} \tag{5}$$

To make the IMC controller proper, it is mandatory to add a filter. Thus, the IMC controller is designed as

$$Q = \tilde{Q} f = P_M^{-1} f \tag{6}$$

To obtain a superior response for integrating processes, the IMC controller should satisfy the following conditions.

If the process  $G_p$  has poles near zero at  $z_1, z_2, \dots, z_m$ , then

- (i) should have zeros at  $z_1, z_2, \dots, z_m$
- (ii) should also have zeros at  $z_1, z_2, \dots, z_m$

Since the IMC controller  $Q$  is designed as  $Q = P_M^{-1} f$ , the first condition is satisfied automatically because  $P_M^{-1}$  is the inverse of the model portion with the poles near zero. The second condition can be fulfilled by designing the IMC filter  $f$  as:

$$f = \frac{\sum_{i=1}^m \beta_i s^i + 1}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} \tag{7}$$

where  $\lambda$  is an adjustable parameter which controls the trade-off between the performance and robustness,  $r$  is selected to be large enough to make the IMC controller (semi-) proper, and  $\beta_i$  is determined by Eq. (8) to cancel the poles near zero in  $G_p$ .

$$1 - G_p Q \Big|_{s=z_1, \dots, z_m} = \left[ 1 - \frac{P_A (\sum_{i=1}^m \beta_i s^i + 1)}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} \right] \Big|_{s=z_1, \dots, z_m} = 0 \tag{8}$$

Then, the IMC controller is described as:

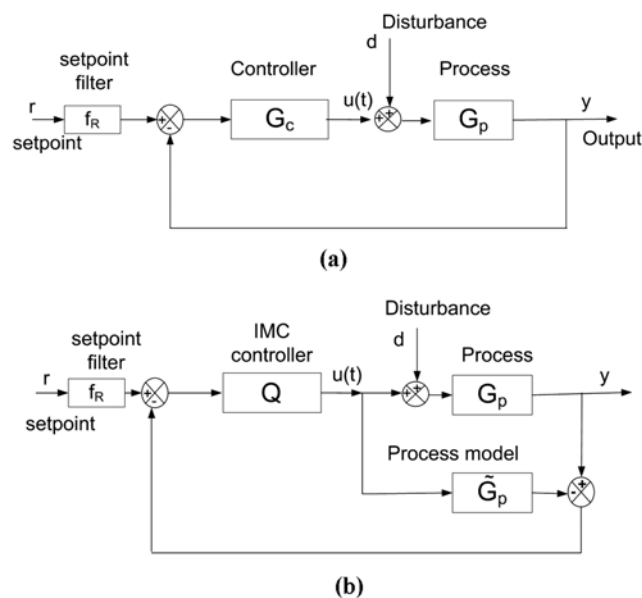
$$Q = P_M^{-1} \frac{(\sum_{i=1}^m \beta_i s^i + 1)}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} \tag{9}$$

Thus, the resulting setpoint and disturbance responses are obtained as:

$$\frac{y}{r} = G_p Q f_r = P_A \frac{(\sum_{i=1}^m \beta_i s^i + 1)}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} f_r \tag{10}$$

$$\frac{y}{d} = (1 - \tilde{G}_p Q) G_p = \left( 1 - P_A \frac{(\sum_{i=1}^m \beta_i s^i + 1)}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} \right) G_p \tag{11}$$

The numerator expression  $(\sum_{i=1}^m \beta_i s^i + 1)$  in Eq. (10) causes an exces-



**Fig. 1. Block diagram of control system (a) classical feedback control structure (b) the IMC structure.**

sive overshoot in the servo response, which can be eliminated by introducing a setpoint filter to compensate for this overshoot.

From the above design procedure, a stable, closed-loop response can be achieved by using the IMC controller. The ideal feedback controller that is equivalent to the IMC controller can be expressed in terms of the internal model  $\tilde{G}_p$  and the IMC controller Q:

$$G_c = \frac{Q}{1 - \tilde{G}_p Q} \quad (12)$$

Substituting Eqs. (4) and (7) into (12) gives the ideal feedback controller:

$$G_c = \frac{p_m^{-1} \frac{(\sum_{i=1}^m \beta_i s^i + 1)}{(\lambda^2 s^2 + 2\lambda \zeta s + 1)^r}}{1 - \frac{p_d(\sum_{i=1}^m \beta_i s^i + 1)}{(\lambda^2 s^2 + 2\lambda \zeta s + 1)^r}} \quad (13)$$

Since the resulting controller does not have a standard PID controller form, the remaining issue is to design the PID controller that resembles the equivalent feedback controller most closely. Lee et al. [15] proposed an efficient method for converting the ideal feedback controller  $G_c$  to a standard PID controller. Since  $G_c$  has an integral term, it can be expressed as

$$G_c = \frac{f(s)}{s} \quad (14)$$

Expanding  $G_c$  in Maclaurin series in gives

$$G_c = \frac{1}{s} \left( f(0) + f'(0)s + \frac{f''(0)}{2}s^2 + \dots \right) \quad (15)$$

The first three terms of the above expansion can be interpreted as the standard PID controller given by

$$G_c = K_c \left( 1 + \frac{1}{\tau_i s} + \tau_D s + \dots \right) \quad (16)$$

where

$$K_c = f'(0) \quad (17a)$$

$$\tau_i = f'(0)/f(0) \quad (17b)$$

$$\tau_D = f''(0)/2f'(0) \quad (17c)$$

### PROPOSED TUNING RULE

#### 1. First Order Delay Integrating Process (FODIP)

So far, most of the design methods associated with an integrating process were intended for integrator/dead time processes for PID controllers and Smith predictors [1-6]. In this paper, an integrating process involving time delay and a time constant is studied, which is further extended to integrating processes with positive and negative zero, and is given by the following transfer function:

$$G_p = \frac{K e^{-\theta s}}{s(\tau s + 1)} \quad (18)$$

The above process is considered as a second order plus dead time (SOPDT) model by approximating it as

$$G_p = \frac{K e^{-\theta s}}{(\tau s + 1)(s + 1/\phi)} = \frac{\phi K e^{-\theta s}}{(\tau s + 1)(\phi s + 1)} \quad (19)$$

where  $\phi$  is an arbitrary constant with a sufficiently large value.

The following IMC filter is utilized

$$f = \frac{(\beta_2 s^2 + \beta_1 s + 1)}{(\lambda^2 s^2 + 2\lambda \zeta s + 1)^2} \quad (20)$$

The resulting IMC controller becomes

$$Q = \frac{(\tau s + 1)(\phi s + 1)(\beta_2 s^2 + \beta_1 s + 1)}{K(\lambda^2 s^2 + 2\lambda \zeta s + 1)^2} \quad (21)$$

Therefore, the ideal feedback controller is obtained as

$$G_c = \frac{(\tau s + 1)(\phi s + 1)(\beta_2 s^2 + \beta_1 s + 1)}{K[(\lambda^2 s^2 + 2\lambda \zeta s + 1)^2 - e^{-\theta s}(\beta_2 s^2 + \beta_1 s + 1)]} \quad (22)$$

The analytical PID formula can be obtained from Eqs. (14)-(17) as:

$$K_c = \frac{\tau_i}{\phi K(4\lambda \zeta + \theta - \beta_1)} \quad (23a)$$

$$\tau_i = (\phi + \tau + \beta_1) - \frac{(-\theta^2/2 + \theta\beta_1 - \beta_2 + 2\lambda^2 + 4\lambda^2 \zeta^2)}{(4\lambda \zeta + \theta - \beta_1)} \quad (23b)$$

$$\tau_D = \frac{(\beta_2 + (\tau + \phi)\beta_1 + \phi\tau) - \frac{(-\theta^3/2 + \beta_1\theta^2/2 + \theta\beta_2 + 4\lambda^3 \zeta)}{(4\lambda \zeta + \theta - \beta_1)}}{\tau_i} - \frac{(-\theta^2/2 + \theta\beta_1 - \beta_2 + 2\lambda^2 + 4\lambda^2 \zeta^2)}{(4\lambda \zeta + \theta - \beta_1)} \quad (23c)$$

The values of  $\beta_1$  and  $\beta_2$  are selected to cancel out the poles at  $-1/\tau$  and  $-1/\phi$ . This requires  $[1 - GQ]_{s=-1/\tau, -1/\phi} = 0$  and thus  $[1 - (\beta_2 s^2 + \beta_1 s + 1)e^{-\theta s}/(\lambda^2 s^2 + 2\lambda \zeta s + 1)^2]_{s=-1/\tau, -1/\phi} = 0$ . The values of  $\beta_1$  and  $\beta_2$  are obtained after simplification and given below.

$$\beta_1 = \frac{\tau^2 \left( \frac{\lambda^2}{\tau^2} - \frac{2\lambda \zeta}{\tau} + 1 \right)^2 e^{-\theta \tau} - \phi^2 \left( \frac{\lambda^2}{\phi^2} - \frac{2\lambda \zeta}{\phi} + 1 \right)^2 e^{-\theta \phi} + (\phi^2 - \tau^2)}{(\phi - \tau)} \quad (24)$$

$$\beta_2 = \tau^2 \left[ \left( \frac{\lambda^2}{\tau^2} - \frac{2\lambda \zeta}{\tau} + 1 \right)^2 e^{-\theta \tau} - 1 \right] + \beta_1 \tau \quad (25)$$

In the IMC filter, the denominator is selected to have a general second order form in the proposed study. In the conventional IMC filter, the value of  $\zeta=1.0$  is used. In this study,  $\zeta$  is a free parameter and depends upon the process.  $\zeta=1.0$  is usually recommended for lag time dominant integrating processes. However, either for an integrating process with a negative zero or for a dead time dominant integrating process, an overdamped  $\zeta$  can be selected to avoid an excessive undershoot in the disturbance rejection response.

A setpoint filter  $f_r$  comprised of the lag term  $(\beta_2 s^2 + \beta_1 s + 1)$  is used to enhance the servo response, by eradicating the excessive overshoot, and is given as:

$$f_r = \frac{\gamma \beta_1 s + 1}{(\beta_2 s^2 + \beta_1 s + 1)} \quad (26)$$

where  $0 \leq \gamma \leq 1$ . The extreme case with  $\gamma=0$  has no lead term in the setpoint filter which would cause a slow servo response. Note that  $\gamma$  can be adjusted online to obtain the desired speed of the setpoint response.

In the simulation study, a widely accepted, weighting-type setpoint filter is used for all methods, except for the proposed method, to

eliminate the overshoot in the setpoint response. This filter is given as:

$$f_R = (\gamma\tau_s + 1) / (\tau\tau_p s^2 + \tau_s + 1) \tag{27}$$

where  $0 \leq \gamma \leq 1$ .

**2. Delay Integrating Process (DIP)**

The commonly used delay integrating process model for chemical industries is given below

$$G_p = \frac{K e^{-\theta s}}{s} \tag{28}$$

The DIP process can be modeled as a first order process plus dead time by approximating it as follows

$$G_p = \frac{K e^{-\theta s}}{s} = \frac{K e^{-\theta s}}{s + 1/\phi} = \frac{\phi K e^{-\theta s}}{\phi s + 1} \tag{29}$$

where  $\phi$  is an arbitrary constant with a sufficiently large value i.e.,  $\phi \gg 1$ . The proposed filter is  $f = (\beta s + 1) / (\lambda^2 s^2 + 2\lambda\zeta s + 1)$  for the DIP model.

Therefore, the resulting IMC controller becomes  $Q = (\phi s + 1) / K\phi(\lambda^2 s^2 + 2\lambda\zeta s + 1)$  and thus the PID parameters can be obtained from Eq. (14)-(17) as

$$K_c = \frac{\tau_i}{\phi K (\theta - \beta + 2\lambda\zeta)} \tag{30a}$$

$$\tau_i = (\phi + \beta) - \frac{(\lambda^2 - \theta^2/2 + \beta\theta)}{(\theta - \beta + 2\lambda\zeta)} \tag{30b}$$

$$\tau_D = \frac{(\phi\beta) - \frac{(\theta^3/6 - \beta\theta^2/2)}{(\theta - \beta + 2\lambda\zeta)}}{\tau_i} - \frac{(\lambda^2 - \theta^2/2 + \beta\theta)}{(\theta - \beta + 2\lambda\zeta)} \tag{30c}$$

$$\beta = \phi \left[ 1 - \frac{(\lambda^2 - 2\lambda\zeta\phi + \phi^2)e^{-\theta/\phi}}{\phi^2} \right] \tag{30d}$$

**SIMULATION STUDY**

This section describes the simulation study conducted on the five different types of representative model of integrating process with time delay. The performance and robustness of the control system were evaluated by using the following indices to ensure a fair comparison.

To evaluate the robustness of a control system, the maximum sensitivity,  $M_s$ , which is defined by  $M_s = \max |1/[1 + G_p G_c(i\omega)]|$ , is used. Since is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point  $(-1, 0)$ , a small

$M_s$  value indicates that the stability margin of the control system is large.  $M_s$  is a well-known robustness measurement item and is used by many researchers [1,3,10,18].

To evaluate the closed-loop performance, two performance indices were considered in the case of both a step setpoint change and a step load disturbance, viz., the integral of the absolute error (IAE) defined by  $IAE = \int_0^{\infty} |e(t)| dt$  and the overshoot which acts as a measure of how much the response exceeds the ultimate value following a step change in the setpoint and/or load disturbance.

To evaluate the usage of the manipulated input values, we compute the TV of the input  $u(t)$ , which is the sum of all of its up and down movements. If we discretize the input signal as a sequence

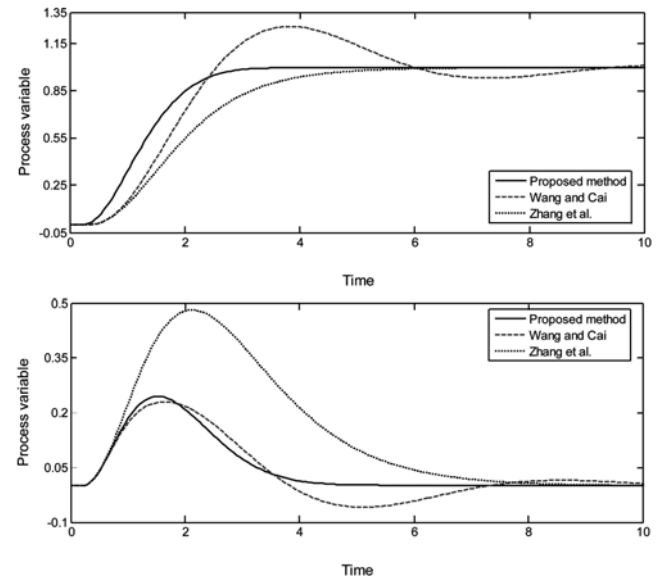
$[u_1, u_2, u_3, \dots, u_i, \dots]$  then  $TV = \sum_{i=1}^{\infty} |u_{i+1} - u_i|$  should be as small as possible. TV is a good measure of the smoothness of a signal [1,3,18].

**Example 1: First Order Delay Integrating Process**

Consider the following integrating process studied by Wang and Cai [8]

$$G_p = \frac{1 e^{-0.2s}}{s(s+1)} \tag{31}$$

The proposed controller was designed by considering the above pro-



**Fig. 2. Response of the nominal system for Example 1.**

**Table 1. Controller parameters and resulting performance indices for Example 1**

Method	$K_c$	$\tau_i$	$\tau_D$	Set-point						Disturbance				
				Nominal case			20% mismatch			Nominal			20% mismatch	
				IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot
Proposed $\lambda=0.433$	3.997	1.893	0.499	1.34	1.0	2.52	1.36	1.0	0.47	0.24	2.0	0.47	0.25	
Wang and Cai	3.094	1.181	0.863	2.35	1.26	2.91	2.10	1.20	0.65	0.23	1.8	0.58	0.23	
Zhang et al. $\lambda=0.597$	2.038	2.991	0.665	2.09	1.0	1.40	2.09	1.0	1.47	1.72	0.48	1.47	0.50	

-Proposed,  $\gamma=0.3$ ,  $f_R = (0.5782s + 1) / (1.0119s^2 + 1.9273s + 1)$

-Wang and Cai,  $\gamma=0$ ,  $f_R = 1 / (1.0199s^2 + 1.1819s + 1)$

-Zhang et al.,  $\gamma=0.3$ ,  $f_R = (0.8973s + 1) / (1.9911s^2 + 2.991s + 1)$ , the extra lag filter  $F_t = 1 / (0.145s + 1)$  in  $G_c = K_c (1 + (1/\tau_s)s + \tau_D s) \cdot F_t$

- $M_s = 1.65$

cess as  $G_p=100e^{-0.2s}/(100s+1)(s+1)$ . The proposed method is compared with two other PID controllers, those of Wang and Cai [8] and Zhang et al. [2]. The controller parameters, including the performance and robustness matrix, are listed in Table 1. In order to ensure a fair comparison, all of the compared controllers are tuned to have  $M_s=1.65$  by adjusting their respective  $\lambda$  values.

A unit step change is introduced in both the setpoint and load disturbance. Fig. 2 compares the setpoint and load disturbance responses obtained by using the three controllers being compared. The 2DOF control scheme using the setpoint filter was used in each method to enhance the setpoint response. The proposed controller shows faster disturbance rejection than those of Wang and Cai [8] and Zhang et al. [2].

Although the overshoot is large in Zhang et al.'s [2] method, Wang and Cai's [8] method shows undershoot and requires a long settling time. The proposed controller shows significant advantages, exhibiting a fast settling time, both in its setpoint tracking and disturbance rejection.

The robust performance is evaluated by simultaneously inserting a perturbation uncertainty of 20% into all three parameters in the worst direction and finding the actual process as  $G_p=1.2e^{-0.24s}/s(0.8s+1)$ . The simulation results for the model mismatch for the three methods are given in Table 1. The performance and robustness indices clearly demonstrate the superior robust performance of the proposed controller.

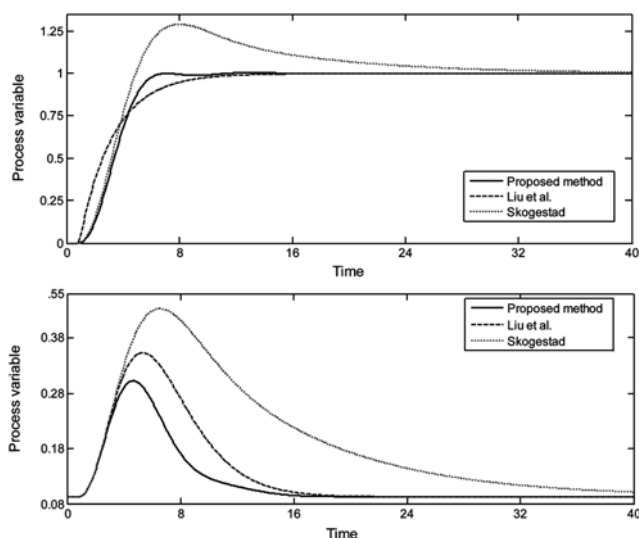
**Example 2: Double Integrating Process with Dead Time**

Consider the double integrating process (Liu et al. [6])

$$G_p = \frac{e^{-0.8}}{s^2} \tag{32}$$

In the simulation study, we compare the proposed PID controller with those of Liu et al. [6] and Skogestad [1]. For the proposed method,  $\lambda=1.25$  is selected, which provides a better disturbance rejection response for both the nominal and model mismatch cases. The PID settings of the Liu et al. [6] and Skogestad [1] methods were obtained from Liu et al.'s paper.

The proposed controller was designed by considering the above process as  $G_p=10000e^{-0.8s}/(100s+1)(100s+1)$ . Fig. 3 shows the closed-loop output response for a unit step change in both the setpoint and load disturbance. The controller setting parameters and performance matrix for both the setpoint and load disturbance response are listed in Table 2. To eliminate the overshoot in the setpoint response, a setpoint filter is used in each method, as listed in Table 2. The proposed controller has a small peak and fast settling time in the dis-



**Fig. 3. Response of the nominal system for Example 2.**

turbance rejection, whereas in the setpoint response it has almost no overshoot and a faster settling time than the other methods. This comparison of the output response and the values of the performance matrices listed in Table 2 confirms the superior performance of the proposed controller.

To investigate the robust performance of the proposed controller, 20% parameter perturbations were assumed in  $K$  and  $\theta$  simultaneously towards the worst-case model mismatch as  $G_p=1.2e^{-0.96s}/s^2$ . The simulation results for the model mismatch for the various methods are also given in Table 2. The robustness performance indices demonstrate the superior robust performance of the proposed controller.

**Example 3: Boiler Steam Drum**

An example of an integrating process that has an inverse response is a boiler steam drum. The level is controlled by manipulating the boiler feed water (BFW) to the drum. The drum is located near the top of the boiler and is connected to it by a large number of tubes. Liquid and vapor water circulate between the drum and the boiler as a result of the difference in density between the liquid in the down-comer pipes leading from the bottom of the drum to the base of the boiler and the vapor/liquid mixture in the riser pipes going up through the boiler and back into the steam drum. Luyben [9] suggested that the transfer function of the boiler steam drum after the identification test could be modeled as an integrating process with dead time

**Table 2. Controller parameters and resulting performance indices for Example 2**

Method	$K_c$	$\tau_i$	$\tau_D$	Set-point						Disturbance				
				Nominal case			20% mismatch			Nominal			20% mismatch	
				IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot
Proposed $\lambda=1.25$	0.351	5.880	2.343	3.50	1.0	0.52	4.07	1.08	1.67	0.313	0.32	1.83	0.359	
Liu et al. $\lambda=1.5$	-	-	-	3.30	1.0	4.26	3.38	1.01	2.63	0.39	0.37	2.65	0.426	
Skogestad $\lambda=0.8$	0.0977	6.40	6.40	6.25	1.29	0.44	5.97	1.24	6.52	0.51	0.22	6.43	0.522	

-Proposed,  $\gamma=0.4$ ,  $f_R=(2.3193s+1)/(13.3587s^2+5.7983s+1)$   
 -Liu et al.,  $R(s)=0.4s/(0.1s+1)$ ,  $F_0=s^2(18.62s^2+6.8s+1)/(1.5s+1)^4$   
 -Skogestad, no setpoint filter is used

and inverse response as follows:

$$G_p = \frac{0.547(-0.418s+1)e^{-0.1s}}{(1.06s+1)s} \quad (33)$$

The inverse response time constant (negative numerator time constant) can be approximated as a time delay such as  $(-\theta_0^{inv}s+1) \approx e^{-\theta_0^{inv}s}$  for the proposed controller design. This is reasonable since an inverse response has a deteriorating effect on the control, which is similar to that of a time delay [1]. Therefore, the above process can be approximated as  $G_p = 0.547e^{-0.518s}/(1.06s+1)s$ .

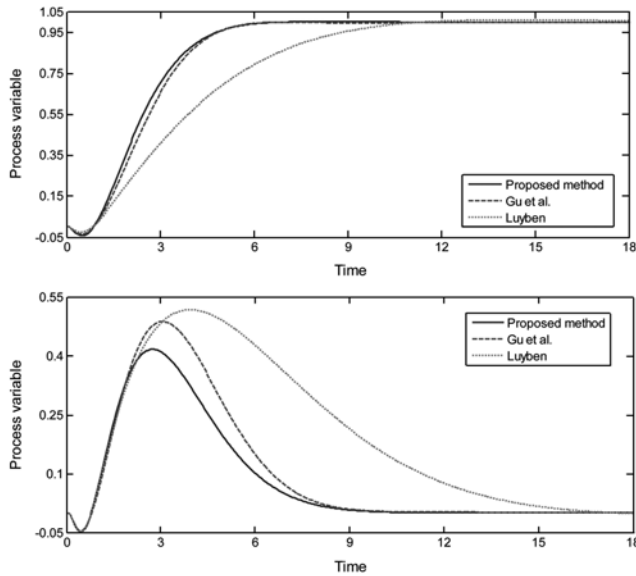


Fig. 4. Response of the nominal system for Example 3.

The PID controllers were designed by using the proposed method, and those of Gu et al. [7] and Luyben [9]. Fig. 4 shows the closed-loop output responses for a unit step change introduced in both the setpoint and load disturbance. In Luyben’s PID tuning method, an integral time of 25% is considered, because it shows a clear advantage in the disturbance rejection over an integral time of 50%. The controller setting parameters are listed in Table 3 including the performance indices. Fig. 4 shows that the proposed method has a smooth and fast response for both the disturbance rejection and setpoint. For the setpoint, the 2DOF controller is used for each tuning method and the setpoint filter is also listed in Table 3. From Fig. 4 and Table 3, it is clear that the proposed method is advantageous over the others.

The robustness of the proposed method was investigated by inserting 10% perturbations into each of the process parameters towards the worst-case model mismatch and assuming the actual process to be  $G_p = 0.6017(-0.4598s+1)e^{-0.11s}/s(0.954s+1)$ . The simulation results for the model mismatch in Table 3 clearly demonstrate the superior robust performance of the proposed controller.

**Example 4:** FODIP with a negative zero

Consider the following process of paper drum dryer cans [10,11].

$$G_p = \frac{0.005(300s+1)e^{-5s}}{s(20s+1)} \quad (34)$$

The PID controller parameter setting for the proposed method and those of Wang and Cluett [11] and Rivera et al. [16] are presented in Table 4. The PID controller settings for the latter two methods were taken from Wang and Cluett [11]. Fig. 5 shows the closed-loop output responses for a unit step change introduced in both the setpoint and load disturbance for these three designs methods.

Wang and Cluett [11] previously demonstrated the superiority of

Table 3. Controller parameters and resulting performance indices for Example 3

Method	$K_c$	$\tau_i$	$\tau_D$	Set-point						Disturbance				
				Nominal case			10% mismatch			Nominal			10% mismatch	
				IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot
Proposed $\lambda=0.7909$	2.437	3.638	0.735	2.52	1.0	1.83	2.49	1.0	1.53	0.43	3.27	1.56	0.430	
Gu et al. $\lambda=0.8$	2.088	3.866	0.688	2.67	1.0	1.72	2.65	1.0	1.89	0.49	2.58	1.92	0.504	
Luyben	1.61	5.75	1.15	4.15	1.01	1.17	4.07	1.0	3.66	0.52	2.28	3.67	0.513	

-Proposed,  $\gamma=0.3$ ,  $f_R=(1.0963s+1)/(2.7528s^2+3.6543s+1)$

-Gu et al.,  $\gamma=0.3$ ,  $f_R=(1.1599s+1)/(2.6597s^2+3.8664s+1)$

-Luyben,  $\gamma=0.3$ ,  $f_R=(1.7250s+1)/(6.6125s^2+5.75s+1)$

Table 4. Controller parameters and resulting performance indices for Example 4

Method	$K_c$	$\tau_i$	$\tau_D$	Set-point						Disturbance				
				Nominal case			10% mismatch			Nominal			10% mismatch	
				IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot
Proposed $\lambda=7.47$	33.87	62.11	14.84	27.04	1.01	1.16	26.08	1.01	14.30	0.48	1.60	14.15	0.626	
Wang and Cluett	1.60	26.0	2.0	14.79	1.03	2.11	12.42	1.06	16.63	0.43	1.16	16.54	0.540	
Rivera et al.	24.80	58.3	13.1	39.0	1.33	1.84	35.35	1.26	21.44	0.55	1.54	21.11	0.687	

-Proposed,  $\gamma=0.6$ ,  $f_R=(36.6554s+1)/(860.9971s^2+61.0924s+1)$ , the extra lag filter  $F_t=1/(300s+1)$  in  $G_c=K_c(1+(1/\tau_i)s+\tau_Ds)\cdot F_t$

-Wang and Cluett, offset of 3% occurs

-Rivera et al., 1DOF controller is used with  $F_t=1/(300s+1)$

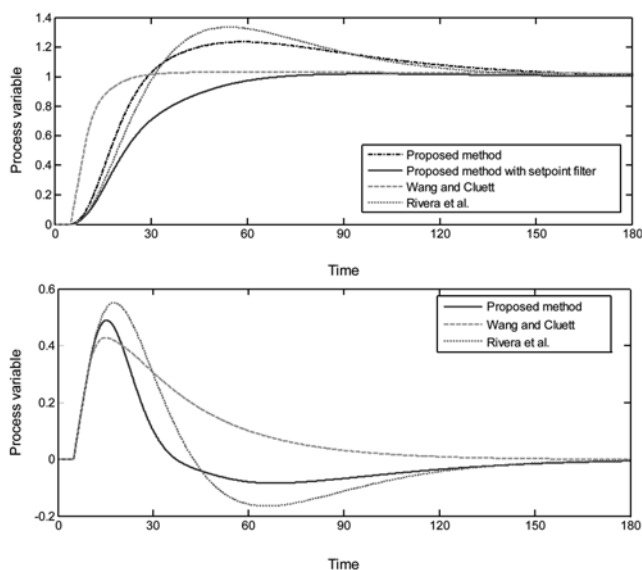


Fig. 5. Response of the nominal system for Example 4.

their method over that of Rivera et al. [16], and Fig. 5 also clearly shows that Rivera et al.'s method has a large overshoot. Although Wang and Cluett's [11] method shows less overshoot for the disturbance rejection, the response is very sluggish and a long settling time is required. In the servo response, Wang and Cluett's method [11] shows both less overshoot and shorter settling time, but there is a steady state offset of 3%. The overshoot in the proposed method can be minimized by using the setpoint filter and the response is also given in Fig. 5. The proposed method shows a clear advantage over the others and exhibits a lower IAE value, as shown in Table 4. In the proposed method, the sharp undershoot in the disturbance rejection response can be minimized by selecting the overdamped IMC filter, and for the above example,  $\zeta=2.0$  gives the minimum IAE value with  $M_s=1.92$ .

The robustness of the controller is evaluated by inserting a perturbation uncertainty of 10% in all four parameters simultaneously to obtain the worst case model mismatch, i.e.,  $G_p=0.0055(330s+1)e^{-5.5s}/s(18s+1)$  as an actual process, whereas the controller settings are those calculated for the process with the nominal model. Table 4 shows both the setpoint tracking and disturbance rejection performance indices for model mismatch as well. The controller settings of the proposed method provide the most robust performance for regulatory problems, whereas Wang and Cluett's method [11]

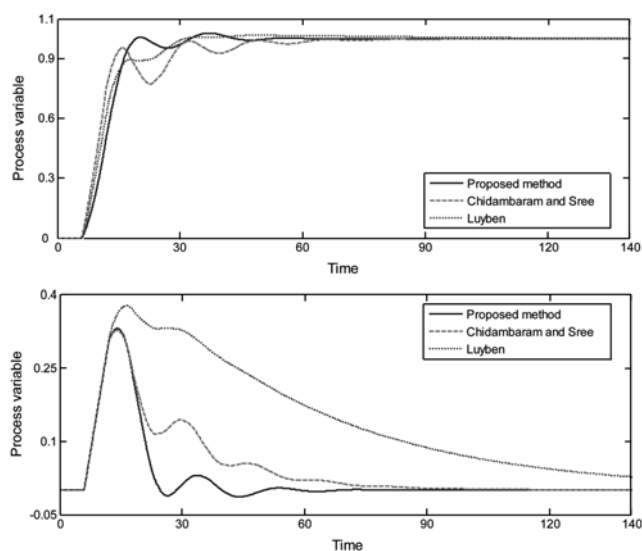


Fig. 6. Response of the nominal system for Example 5.

has the best setpoint response with offset.

**Example 5:** DIP Process

The following DIP model [13,14] was considered to demonstrate the superiority of the proposed method.

$$G_p = \frac{0.0506e^{-6s}}{s} \tag{35}$$

The methods of Luyben [13] and Chidambaram and Sree [14] and the proposed method were used to design a PID controller, and the parameter settings for each method are listed in Table 5, which were obtained from the respective papers.

The output responses for a unit step change introduced in both the setpoint and load disturbance are given in Fig. 6. The setpoint filter is used in each method to reduce the overshoot in the servo response. The disturbance rejection of the proposed controller is fast and it requires less time to settle at a steady state value, while Luyben's method [13] is the slowest. Fig. 6 and Table 5 reveal that the disturbance rejection and setpoint response for the proposed controller are superior to those of the other tuning methods.

The robustness of the controllers is also evaluated by inserting a perturbation uncertainty of 10% both in the gain and the dead time simultaneously towards the worst case model mismatch, such that  $G_p=0.0557e^{-6.6s}/s$ . The simulation results for the plant-model mismatch are also given in Table 5. It seems that the proposed method

Table 5. Controller parameters and resulting performance indices for Example 5

Method	$K_c$	$\tau_i$	$\tau_d$	Set-point						Disturbance				
				Nominal case			10% mismatch			Nominal			10% mismatch	
				IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot	TV	IAE	overshoot
Proposed $\lambda=3.437$	4.282	14.91	2.34	12.65	1.02	6.54	17.65	1.15	3.76	0.33	4.52	7.75	0.38	
Chidambaram and Sree	4.066	27.0	2.70	13.48	1.0	10.72	17.52	1.12	6.63	0.33	4.75	7.22	0.38	
Luyben	2.564	56.32	3.56	13.34	1.02	5.37	12.54	1.03	21.0	0.38	1.78	21.0	0.43	

-Proposed,  $f_R=1/(12.186s+1)$

-Chidambaram and Sree,  $\gamma=0.5$ ,  $f_R=(13.5s+1)/(72.9s^2+27s+1)$

-Luyben,  $\gamma=0.8$ ,  $f_R=(45.056s+1)/(200.555s^2+56.32s+1)$ ,  $F_t=1/(0.382s+1)$

and that of Chidambaram and Sree [14] offer similar performance, even for severe process uncertainties.

**1. Beneficial range of the Proposed Method**

The proposed PID controller has a clear advantage over the other well-known methods (Zhang et al. [2] and Chen and Seborg [3]).

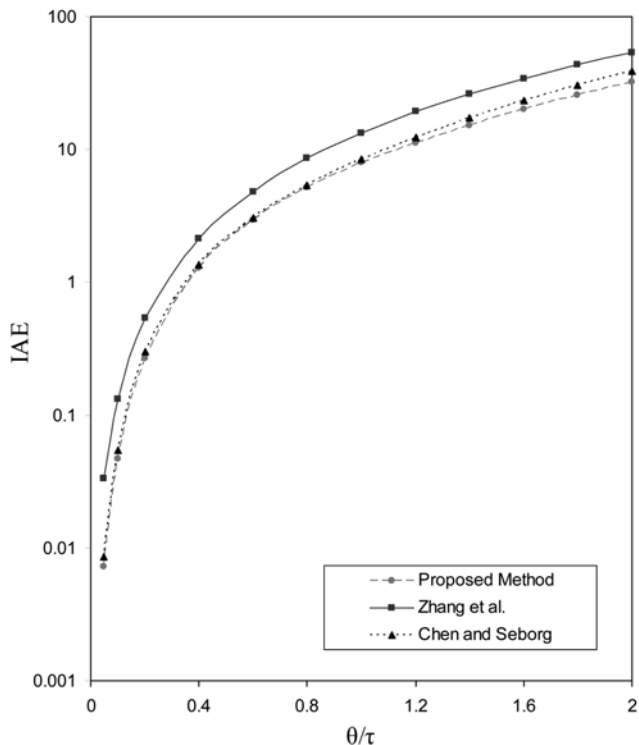


Fig. 7. Performance of the proposed controller vs. other conventional controllers.

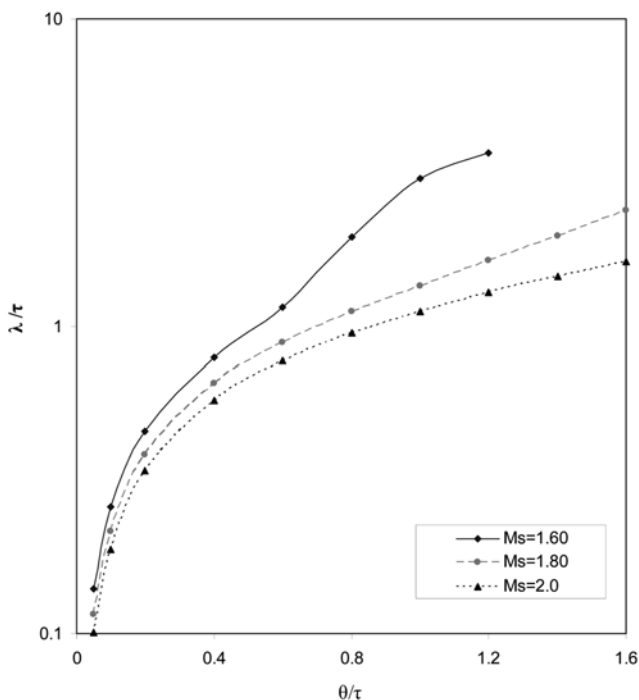


Fig. 8.  $\lambda$  guidelines for FODIP.

Fig. 7 compares the IAE values of the disturbance rejection for various dead time to lag time ratios for the FODIP model used in Example 1 (by changing  $\theta$  while keeping  $\tau$  fixed).  $\lambda$  is chosen so that  $M_s=2.0$  in each case. As shown in the figure, the proposed PID controller gives a smaller IAE value than the other controllers over a broad range of  $\theta/\tau$  ratios. From Fig. 7, it is clear that the proposed method offers consistently improved performance.

**2.  $\lambda$  Guideline**

In the proposed tuning rule, the closed-loop time constant  $\lambda$  governs the tradeoff between the robustness and performance of the control system. As  $\lambda$  decreases, the closed-loop response becomes faster and can become unstable. On the other hand, as  $\lambda$  increases, the closed-loop response becomes sluggish and more stable. A good tradeoff is obtained by choosing  $\lambda$  so as to give an  $M_s$  value in the range of 1.6-2.0. The  $\lambda$  guideline plot for FODIP at several robustness levels is shown in Fig. 8. For the DIP process, the  $\lambda$  guidelines are plotted in Fig. 9. As shown in the figure, the resulting plot is almost a straight line and the desired  $\lambda$  value can also be obtained from the linear relation given by  $\lambda = \alpha\theta$ , where  $\alpha=2.245, 1.5757$  and  $1.221$  for  $M_s=1.6, 1.8$ , and  $2.0$ , respectively, for the DIP process.

**CONCLUSIONS**

In this article, we discussed an IMC-based PID controller design method for several types of integrating process with time delay. Several important representative processes were considered in the simulation study, in order to demonstrate the superiority of the proposed method. The design method was based on the disturbance rejection and a setpoint filter was suggested to eliminate the overshoot

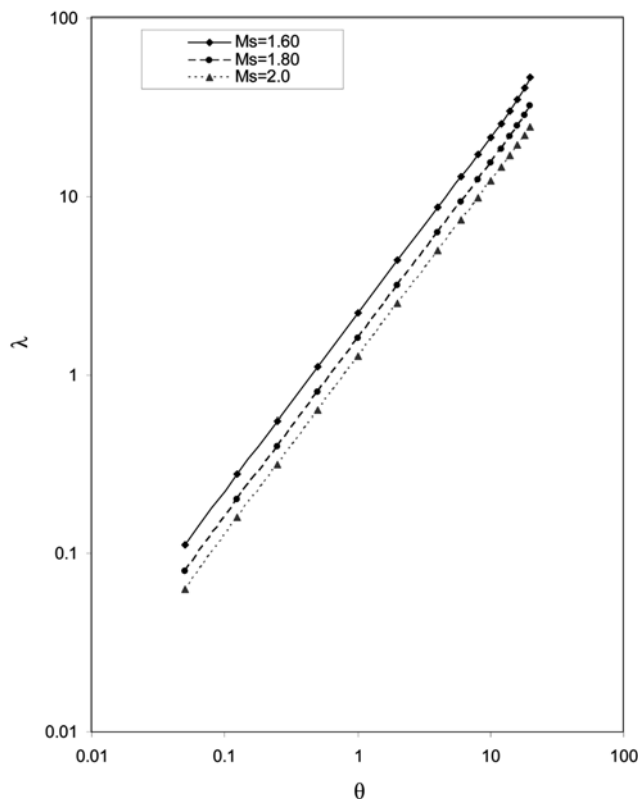


Fig. 9.  $\lambda$  guidelines for DIP.



in the setpoint response. The results showed that both the nominal and robustness performances of the PID controller were significantly enhanced in the proposed method. The proposed controller consistently achieved superior performance for several process classes. In the robustness study conducted by simultaneously inserting a perturbation uncertainty in all parameters in order to obtain the worst-case model mismatch, the proposed method was found to be superior to the other methods. A guideline was suggested for the selection of  $\lambda$  for different robustness levels by evaluating the Ms value over a wide range of  $\theta/\tau$  ratios. An IAE comparison was also performed at a fixed Ms for several tuning methods, which clearly indicated that the proposed method gives consistently better performance over a broad range of  $\theta/\tau$  ratios.

#### ACKNOWLEDGMENT

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