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Enhanced disturbance rejection for open-loop unstable process with time delay

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ABSTRACT

The present study suggests a disturbance estimator design method for application to a recently published, two-degree-of-freedom, control scheme for open-loop, unstable processes with time delay. A simple PID controller cascaded with a lead–lag filter replaces the high-order disturbance estimator for enhanced performance. A new analytical method on the basis of the IMC design principle, featuring only one user-defined tuning parameter, is developed for the design of the disturbance estimator. Several illustrative examples taken from previous works are included to demonstrate the superiority of the proposed disturbance estimator. The results confirm the superior performance of the proposed disturbance estimator in both nominal and robust cases. The proposed method also offers several important advantages for industrial process engineers: it covers several classes of unstable process with time delay in a unified manner, and is simple and easy to design and tune.

1. Introduction

Time-delayed unstable processes are commonly encountered in process industries and are difficult to control due to right-half plane (RHP) poles. A time delay is introduced into the transfer function description of such system due to the measurement delay or an actuator delay or by the approximation of higher dynamics of the system by that of a lower order plus delay systems.

The design area of the control of an unstable delay system has recently attracted much research attention [1–6]. Proportional-Integrative-Derivative (PID) controllers are the most common and successful type found in industrial control applications due to their simple structure and meaning of the corresponding three parameters, which simplifies the understanding of the PID control compared to most other advanced control techniques. In addition, the PID controller usually exhibits satisfactory performance in many situations. The widespread use of PID controllers raises the importance of designing a simple but efficient method for tuning the controller.

The distinguishable framework for control system design and implementation of the internal model control (IMC) (Morari and Zafiriou, [1]; Lee et al., [2]; Yang et al., [5]; Shamsuzzoha and Lee, [6]) has directed much research attention on exploiting the IMC principle for the design of equivalent feedback controllers for unstable processes. The principle has attracted the attention of industrial users because there is only one user-defined tuning parameter, which is directly related to the closed-loop time constant.

Due to its internal instability, the IMC design principle cannot be directly applied for the design of the unstable processes. The several modified, IMC-based control strategies have therefore been developed for controlling unstable processes with time delay and are freely available in the literature [2,5–8]. Furthermore, two-degree-of-freedom (2DOF) control methods based on the Smith–Predictor (SP) were proposed (Kwak et al. [9]; Majhi and Atherton [10]; Zhang et al. [11]) to achieve a smooth nominal setpoint response without overshoot for first-order unstable processes with time delay. Kwak et al. [9] have proposed a modified SP for unstable processes, which predicts the dynamics of the actual process from the process model. However, the method is sensitive to uncertainties in the process parameters. Majhi and Atherton [10] have modified the original SP for unstable time delay process, proposed a PD controller for stabilizing the unstable process and obtained improved performance with the added advantage of decoupling the disturbance rejection response from the setpoint tracking. Zhang et al. [11] have also suggested a modified SP structure for unstable, first-order, time delay process. They modified the structure proposed by Kwak et al. [9] and designed a PID controller to stabilize the unstable process and reject the disturbance based on the desired closed-loop response.

It is important to note that in the several design methods based on either the modified IMC or the modified SP methods, the nominal setpoint response tends to be faster without overshoot for unstable processes. In fact, the design methods based on the modified IMC and SP methods employ the nominal process model in their control structures, which accounts for their good performance. However, many existing 2DOF control methods are

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restricted to the unstable processes in the form of a first-order rational part plus time delay, which cannot, in fact, represent a variety of industrial and chemical unstable processes well enough. Besides, the usual presence of unmodeled dynamics inevitably tends to deteriorate the control system performance.

Recently Liu et al. [8] proposed a novel, 2DOF control scheme with an analytical design method, as shown in Fig. 1, where the process model G_m is divided in two parts: a delay-free part G_{mo} with a dead time of $e^{-\theta_m s}$, i.e., $G_m = G_{mo}e^{-\theta_m s}$, and a controller C that is responsible for the setpoint tracking. A disturbance estimator F is used for load disturbances rejection and a controller G_c for stabilizing the setpoint response.

The design of G_c enables it to contribute as a P or PD controller and converts the system into an open-loop for setpoint tracking. The analytical design procedure for both C and F is developed based on the H_2 optimal performance objective and available in details in Liu et al. [8]. It is equivalent to the integral-squared-error (ISE) performance specification. Liu et al.'s [8] method provides the clear advantage of remarkably improving the reference tracking and both the nominal setpoint and load disturbance response are decoupled from each other. The controller C and F is easily tuned by monotonously increasing or decreasing the adjustable parameter so that it can cope with the process uncertainty in practice and thereby give the best compromise between the nominal system performance and its robust stability.

Although Liu et al.'s [8] method provides satisfactory setpoint tracking performance, the disturbance estimator is somewhat complicated to design and has a limited performance. Furthermore, disturbance rejection in process industries is commonly much more important than setpoint tracking for many control applications. The high-order controller form they proposed for the disturbance estimator has not gained much popularity in process industries yet and is also difficult to implement.

The objective of the present study is to develop a unified and simple design method for a disturbance estimator to obtain robust and enhanced disturbance rejection performance in the 2DOF control scheme by Liu et al. [8]. The proposed method provides the disturbance estimator in PID form in series with a lead–lag filter as follows

$$F = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\frac{1 + \alpha s}{1 + \beta s}. \quad (1)$$

The controller type given in Eq. (1) is popular to process industries and easy to implement in the modern control hardware as well.

2. Controller design procedure

As seen in Fig. 1, the process model is related as:

$$G'_m = \frac{y_m}{r_f} = \frac{G_{mo}e^{-\theta_m s}}{1 + G_c G_{mo}}. \quad (2)$$

Consequently, the setpoint transfer function is given by:

$$H_r = \frac{y}{r} = \frac{CG_p}{1 + G_c G_{mo}} \cdot \frac{1 + FG_{mo}e^{-\theta_m s}}{1 + FG_p}. \quad (3)$$

In the nominal case (i.e., $G_m = G_p$), therefore, the setpoint transfer function is simplified as:

$$H_r = \frac{y}{r} = \frac{CG_p}{1 + G_c G_{mo}}. \quad (4)$$

Since the dead-time term is discarded in the characteristic equation of the nominal setpoint transfer function, it certainly contributes to achieving a smooth servo response. The tuning formulae for G_c and C for several processes are listed in Table 1 and the detail design procedure is available in literature Liu et al. [8].

Table 1
Summary of setpoint controller design by Liu et al. [8].

Processes	G_c	$C(s)$
$\frac{ke^{-\theta s}}{(\tau_1 s-1)(\tau_2 s-1)}$	$k_d s$	$\frac{\tau_1\tau_2 s^2+(k_d k-\tau_1-\tau_2)s+1}{k(\lambda_c s+1)^2}$
$\frac{ke^{-\theta s}}{(\tau_1 s-1)(\tau_2 s+1)}$	k_c, if $\tau_1 > \tau_2$	$\frac{\tau_1\tau_2 s^2+(\tau_1-\tau_2)s+k_c k-1}{k(\lambda_c s+1)^2}$
	$k_c + k_d s$, if $\tau_1 \le \tau_2$	$\frac{\tau_1\tau_2 s^2+(k_d k+\tau_1-\tau_2)s+k_c k-1}{k(\lambda_c s+1)^2}$
$\frac{ke^{-\theta s}}{s(\tau s-1)} = \frac{k\phi e^{-\theta s}}{(\phi s-1)(\tau s-1)}$	$k_c + k_d s$	$\frac{\tau s^2+(k_d k-1)s+k_c k}{k(\lambda_c s+1)^2}$

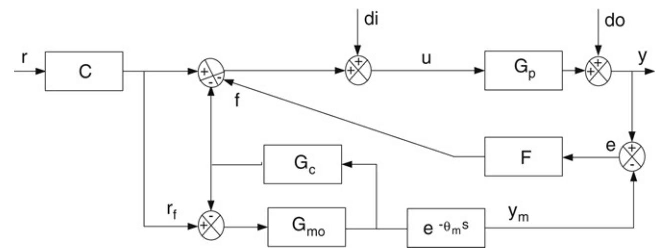

Fig. 1. Two-degree-of-freedom control structure by Liu et al. [8].

Past research experience has demonstrated that disturbance rejection is much more important than setpoint tracking in many process control applications. Therefore, we have emphasized the design of the disturbance estimator for improved disturbance rejection. The closed-loop transfer function for disturbance rejection is given by:

$$H_{di} = \frac{y_{di}}{di} = \frac{G_p}{1 + FG_p} \quad (5)$$

$$H_{do} = \frac{y_{do}}{do} = \frac{1}{1 + FG_p}. \quad (6)$$

It is important to mention that in Fig. 1, controller F is responsible for the disturbance rejection. In the proposed study the IMC design principle utilized to design the disturbance estimator controller F for enhanced performance.

To obtain a superior response for unstable processes, if the process model, G_p, has unstable poles, $up_1, up_2, \ldots, up_m$, then the IMC controller Q should have zeros at $up_1, up_2, \ldots, up_m$ and also $1 - G_pQ$ should have zeros at $up_1, up_2, \ldots, up_m$.

Since the IMC controller Q is designed as $Q = p_m^{-1}f$ ($G_m = p_m p_A$, where p_m and p_A are the portions of the model inverted and not inverted by the controller, respectively, and $p_A(0) = 1$), the first condition is satisfied automatically because p_m^{-1} is the inverse of the model portion with the unstable poles or poles near zero. The second condition can be fulfilled by designing the IMC filter f as:

$$f = \frac{\sum_{i=1}^{m} a_i s^i + 1}{(\lambda_f s + 1)^r} \quad (7)$$

where λ_f is an adjustable parameter which controls the trade-off between the performance and robustness, r is selected to be large enough to make the IMC controller (semi-)proper, and a_i is determined by Eq. (8) to cancel the unstable poles in G_p.

$$1 - G_pQ\big|_{s=z_1,\ldots z_m} = \left|1 - \frac{p_A(\sum_{i=1}^{m} a_i s^i + 1)}{(\lambda_f s + 1)^r}\right|_{s=z_1,\ldots,z_m} = 0. \quad (8)$$

Then, the IMC controller is described as:

$$Q = p_m^{-1}\frac{\left(\sum_{i=1}^{m} a_i s^i + 1\right)}{(\lambda_f s + 1)^r}. \quad (9)$$

From the above design procedure, a stable, closed-loop response can be achieved by using the IMC controller. The ideal feedback controller that is equivalent to the IMC controller can be expressed in terms of the internal model G_m and the IMC controller Q:

$$F_{im}(s) = \frac{Q}{1 - G_m Q}. \tag{10}$$

Substituting the value of the process model G_m and the IMC controller Q into Eq. (10) gives the ideal feedback controller:

$$F_{im}(s) = \frac{p_m^{-1}\frac{\left(\sum_{i=1}^{m} a_i s^i + 1\right)}{(\lambda_f s+1)^r}}{1 - \frac{p_A\left(\sum_{i=1}^{m} a_i s^i + 1\right)}{(\lambda_f s+1)^r}}. \tag{11}$$

Let us consider the design of the following unstable process with two RHP poles as a representative model:

$$G_p = \frac{k e^{-\theta s}}{(\tau_1 s - 1)(\tau_2 s - 1)} \tag{12}$$

where k is the gain, τ_1 and τ_2 are the time constants, and θ is the time delay. The IMC filter structure is chosen as

$$f = \frac{\left(a_2 s^2 + a_1 s + 1\right)}{\left(\lambda_f s + 1\right)^4}. \tag{13}$$

The IMC filter form in Eq. (13) was also utilized by several researchers [2,12] to design the IMC–PID controller. The resulting IMC controller becomes

$$Q = \frac{(\tau_1 s - 1)(\tau_2 s - 1)\left(a_2 s^2 + a_1 s + 1\right)}{K\left(\lambda_f s + 1\right)^4}. \tag{14}$$

Therefore, the ideal desired disturbance estimator obtained as:

$$F_{im}(s) = \frac{(\tau_1 s - 1)(\tau_2 s - 1)\left(a_2 s^2 + a_1 s + 1\right)}{k\left[\left(\lambda_f s + 1\right)^4 - e^{-\theta s}\left(a_2 s^2 + a_1 s + 1\right)\right]}. \tag{15}$$

This ideal disturbance estimator can be approximated to a simple PID cascaded with a lead–lag filter. The dead time $e^{-\theta s}$ in Eq. (15) is approximated by 1/1 Pade expansion,

$$e^{-\theta s} = \frac{(1 - \theta s/2)}{(1 + \theta s/2)} \tag{16}$$

which results in $F_{im}(s)$ given in Box I.

Box I reveals that the resulting disturbance estimator has the form of a PID controller cascaded with a high-order filter. The analytical PID formula can be obtained by rearranging the equation in Box I as below:

$$K_C = \frac{a_1}{k\left(4\lambda_f + \theta - a_1\right)}; \qquad \tau_I = a_1; \qquad \tau_D = \frac{a_2}{a_1}. \tag{17}$$

Furthermore, it is obvious from Eq. (8) that the remaining part of the denominator in Box I contains the factor of the process poles, $(\tau_1 s - 1)(\tau_2 s - 1)$. Therefore, the parameter β can be obtained by taking the first derivative of the equation given in Box II and substituting $s = 0$ as:

$$\beta = \frac{\left(a_1\theta/2 - a_2 + 2\lambda_f\theta + 6\lambda_f^2\right)}{\left(\theta + 4\lambda_f - a_1\right)} + (\tau_1 + \tau_2). \tag{18}$$

The filter parameter α can be easily obtained from Box I:

$$\alpha = 0.5\theta. \tag{19}$$

Since the high-order γs^2 term has little impact on the overall control performance in the control relevant frequency range, the remaining part of the fraction in Box II can be successfully approximated to a simple first-order, lead–lag filter, as $(1 + \alpha s)/(1 + \beta s)$.

The values of a_1 and a_2 are selected to cancel out the poles at $1/\tau_1$ and $1/\tau_2$. This requires $\left[1 - G_p Q\right]\big|_{s=1/\tau_1,1/\tau_2} = 0$ and thus $\left[1 - \left(a_2 s^2 + a_1 s + 1\right)e^{-\theta s}/\left(\lambda_f s + 1\right)^4\right]\Big|_{s=1/\tau_1,1/\tau_2} = 0$. The values of a_1 and a_2 are obtained after simplification and given below.

$$a_1 = \frac{\tau_1^2\left(\frac{\lambda_f}{\tau_1} + 1\right)^4 e^{\theta/\tau_1} - \tau_2^2\left(\frac{\lambda_f}{\tau_2} + 1\right)^4 e^{\theta/\tau_2} + \left(\tau_2^2 - \tau_1^2\right)}{(\tau_1 - \tau_2)} \tag{20}$$

$$a_2 = \tau_1^2\left[\left(\frac{\lambda_f}{\tau_1} + 1\right)^4 e^{\theta/\tau_1} - 1\right] - a_1\tau_1. \tag{21}$$

Table 2 summarizes the proposed disturbance estimator design. The proposed method can also be applied to other types of second-order unstable process by simply transforming it into the standard form of Eq. (12). For example, the integrating first-order unstable process is approximated with a second-order unstable process as follows:

$$G_p = \frac{k e^{-\theta s}}{s(\tau s - 1)} = \frac{k\phi e^{-\theta s}}{(\phi s - 1)(\tau s - 1)} \tag{22}$$

where ϕ is an arbitrary constant with a sufficiently large value.

Remark 1. For a second-order unstable process without any zero, it is observed that the designed value of β is too large to provide robust performance when the parametric uncertainties are large. On the basis of extensive simulation study conducted on different second-order unstable processes, the use of 0.1β instead of β ensures robust control performance. It is important to mention that recently Shamsuzzoha et al. [12] and Rao and Chidambaram [13] also recognized the above finding for getting the robust performance for second-order unstable processes.

It is worthwhile to mention that all the IMC approaches utilize some kind of model reduction techniques to convert the ideal feedback controller to the low order PID controller, an approximation error necessarily occurs. The performance of the resulting controller is determined by many design factors such as the type of IMC filter, the order and structure of the low order controller, and the model reduction technique applied. A properly designed PID controller only can mitigate the conversion error remarkably and provide a satisfactory performance enhancement.

3. Robust stability for closed-loop system

The robustness of the proposed controller design can be analyzed theoretically by utilizing the robust stability theorem [1]. It is of significance in the proposed tuning rule to establish the relationship with a multiplicative uncertainty bound and an adjustable parameter λ_f. In the proposed tuning rule, λ_f can be adjusted for the uncertainties in the process and for the load disturbance.

Robust Stability Theorem (Morari and Zafirou, [1]): Let us assume that all plants G_p in the family $\prod$

$$\prod = \left\{G_p : \left|\frac{G_p(i\omega) - G_m(i\omega)}{G_m(i\omega)}\right| < \bar{\ell}_m(\omega)\right\} \tag{23}$$

have the same number of RHP poles and that a particular controller F stabilizes the nominal plant G_m. Then, the system is robustly

$$F_{im}(s)=\frac{[(\tau_1 s-1)(\tau_2 s-1)]\left(a_2s^2+a_1s+1\right)(1+\theta s/2)}{k\left(\theta+4\lambda_f-a_1\right)s\left[1+\frac{\left(a_1\theta/2-a_2+2\lambda_f\theta+6\lambda_f^2\right)}{(\theta+4\lambda_f-a_1)}s+\frac{\left(a_2\theta^2/2+3\lambda_f^2\theta+4\lambda_f^3\right)}{(\theta+4\lambda_f-a_1)}s^2+\frac{\left(2\lambda_f^3\theta+\lambda_f^4\right)}{(\theta+4\lambda_f-a_1)}s^3+\frac{\lambda_f^4\theta^2/2}{(\theta+4\lambda_f-a_1)}s^4\right]}$$

Box I.

$$\left(\gamma s^2+\beta s+1\right)=\frac{\left[1+\frac{\left(a_1\theta/2-a_2+2\lambda_f\theta+6\lambda_f^2\right)}{(\theta+4\lambda_f-a_1)}s+\frac{\left(a_2\theta^2/2+3\lambda_f^2\theta+4\lambda_f^3\right)}{(\theta+4\lambda_f-a_1)}s^2+\frac{\left(2\lambda_f^3\theta+\lambda_f^4\right)}{(\theta+4\lambda_f-a_1)}s^3+\frac{\lambda_f^4\theta^2/2}{(\theta+4\lambda_f-a_1)}s^4\right]}{(\tau_1 s-1)(\tau_2 s-1)}$$

Box II.

Table 2
Summary of the proposed disturbance estimator design.

Process	Disturbance estimator parameters	a_1 and a_2
$\frac{ke^{-\theta s}}{(\tau_1 s-1)(\tau_2 s-1)}$	$K_C=\frac{a_1}{k(4\lambda_f+\theta-a_1)}$; $\tau_I=a_1$; $\tau_D=\frac{a_2}{a_1}$ $\alpha=0.5\theta$ $\beta=\frac{\left(a_1\theta/2-a_2+2\lambda_f\theta+6\lambda_f^2\right)}{(\theta+4\lambda_f-a_1)}+(\tau_1+\tau_2)$	$a_1=\frac{\tau_1^2\left(\frac{\lambda_f}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-\tau_2^2\left(\frac{\lambda_f}{\tau_2}+1\right)^4 e^{\theta/\tau_2}+\left(\tau_2^2-\tau_1^2\right)}{(\tau_1-\tau_2)}$ $a_2=\tau_1^2\left[\left(\frac{\lambda_f}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-1\right]-a_1\tau_1$

stable with the controller F if and only if the complementary sensitivity function $\tilde{\eta}$ for the nominal plant G_m satisfies the following bound:

$$\left\|\tilde{\eta}\bar{\ell}_m\right\|_\infty=\sup_\omega\left|\tilde{\eta}\bar{\ell}_m(\omega)\right|<1. \tag{24}$$

Since $\tilde{\eta}=G_mQ=G_mp_m^{-1}f$ for the IMC controller, the resulting Eq. (24) becomes:

$$\left|G_mp_m^{-1}f\bar{\ell}_m(\omega)\right|<1\quad\forall\,\omega. \tag{25}$$

Thus, the above theorem can be interpreted as $\left|\bar{\ell}_m\right|<1/|\tilde{\eta}|=1/\left|G_mp_m^{-1}f\right|$, which guarantees robust stability when the multiplicative model error is bounded by $|\Delta_m(s)|\le\bar{\ell}_m$.

$$\|\tilde{\eta}(s)\,\Delta_m(s)\|_\infty<1 \tag{26}$$

where $\Delta_m(s)$ defines the process multiplicative uncertainty bound as follows

$$\Delta_m(s)=\left(G_p-G_m\right)/G_m. \tag{27}$$

This uncertainty bound can be utilized to represent the model reduction error, process input actuator uncertainty, and process output sensor uncertainty, etc., which are very frequent in actual process plants.

For the process with two unstable poles, the complementary sensitivity function $\tilde{\eta}(s)$ can be obtained as

$$\tilde{\eta}(s)=\frac{\left(a_2s^2+a_1s+1\right)e^{-\theta s}}{\left(\lambda_f s+1\right)^4}. \tag{28}$$

Substituting Eqs. (20), (21), (27) and (28) into Eq. (26) yields the robust stability constraint required for tuning the adjustable parameters λ_f as given in Box III.

Suppose the process represented by Eq. (12) has uncertainty in different process parameters, i.e., θ, τ, and k.

Consequently, we have to consider the uncertainty in the different parameters separately. Let us consider the process with two unstable poles having the uncertainty in all three parameters as

$$G_p=\frac{(k+\Delta k)e^{-(\theta+\Delta\theta)s}}{(\tau_1 s-1)(\tau_2 s-1)(\Delta\tau s+1)} \tag{29}$$

where Δ is the uncertainty on each process parameter, with the possibility of assuming negative or positive values. It is most common practice that a low order model is approximated from a high order process in the real process plant. Therefore, it is assumed for the uncertainty in the time constant that a small time constant $\Delta\tau$ is neglected/missing in developing the nominal model as considered in Eq. (29). Then the process multiplicative uncertainty bound becomes

$$\Delta_m(s)=\frac{\left(1+\frac{\Delta k}{k}\right)e^{-\Delta\theta s}}{(\Delta\tau s+1)}-1. \tag{30}$$

Substituting $s=i\omega$ into Box III and the uncertainty bound given in Eq. (30) results in the equation given in Box IV.

The robust stability constraint in Box IV is very useful to adjust λ_f where there is uncertainty in the process parameters. It can also be useful for the determination of the maximum allowable values of uncertainty in $\pm\Delta k$, $\pm\Delta\theta$ and $\pm\Delta\tau$ or various combinations of them for which robust stability can be guaranteed. For an example, a plot of $\left|\tilde{\eta}(\omega)\,\bar{\ell}_m(\omega)\right|$ vs. ω can be constructed for a small value of any parametric uncertainty and/or combination of different uncertainties.

It is important to describe that the closed-loop system is robustly stable and has robust performance for load disturbance rejection if and only if the following constraint has to be met [1], i.e.

$$|\Delta_m\tilde{\eta}|+\left|W_1\left[1-\tilde{\eta}\right]\right|<1. \tag{31}$$

For the IMC controller, Eq. (31) becomes:

$$\left|\Delta_mG_mp_m^{-1}f\right|+\left|W_1\left[1-G_mp_m^{-1}f\right]\right|<1 \tag{32}$$

where W_1 is a weight function of the closed-loop sensitivity function, which is given by $(1-\tilde{\eta})$ and usually can be chosen as $1/s$ for the step change of the load disturbance.

The similar stability and robustness analysis can be carried out for the remaining unstable process with one stable pole and integrating and unstable process with dead time.

4. Simulation results

4.1. Maximum sensitivity (Ms) to modeling error

To evaluate the robustness of a control system, the maximum sensitivity, Ms, defined as $Ms=\max\left|1/[1+G_pG_c(i\omega)]\right|$, is used. Since Ms is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point $(-1,0)$,

$$\left\| \frac{\left[\tau_1^2\left[\left(\frac{\lambda_f}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-1\right]-a_1\tau_1\right]s^2+\left[\frac{\tau_1^2\left(\frac{\lambda_f}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-\tau_2^2\left(\frac{\lambda_f}{\tau_2}+1\right)^4 e^{\theta/\tau_2}+\left(\tau_2^2-\tau_1^2\right)}{(\tau_1-\tau_2)}\right]s+1}{\left(\lambda_f s+1\right)^4} \right\|_\infty < \frac{1}{\|\Delta_m(s)\|_\infty}$$

Box III.

$$\frac{\sqrt{\left[\tau_1^2\left[\left(\frac{\lambda_f}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-1\right]-a_1\tau_1\right]^2\omega^2+\left(1-\left[\frac{\tau_1^2\left(\frac{\lambda_f}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-\tau_2^2\left(\frac{\lambda_f}{\tau_2}+1\right)^4 e^{\theta/\tau_2}+\left(\tau_2^2-\tau_1^2\right)}{(\tau_1-\tau_2)}\right]\omega^2\right)^2}}{\left(\lambda_f^2\omega^2+1\right)^2} < \frac{1}{\left|\frac{\left(1+\frac{\Delta k}{k}\right)e^{-\Delta\theta i\omega}}{(\Delta\tau i\omega+1)}-1\right|}$$

$\forall\, \omega > 0$

Box IV.

a small Ms value indicates that the control system has a large stability margin. Ms is a well-known robustness measure that has been used by many researchers (Shamsuzzoha and Lee, [6]; Skogestad, [14]).

4.2. Total variation (TV)

To evaluate the manipulated input usage, we compute the total variation (TV) of the input $u(t)$ which is the sum of all its moves up and down. If we discretize the input signal as the sequence $[u_1, u_2, u_3, \ldots, u_i, \ldots]$, then $TV = \sum_{i=1}^{\infty} |u_{i+1} - u_i|$ should be minimized. TV is a good measure of the control effort (Shamsuzzoha and Lee, [6]; Skogestad, [14]).

Example 1 (*Process with Two Unstable Poles*). Consider the process with two unstable poles, as studied by Liu et al. [8] and Tan et al. [7].

$$G_p = \frac{2e^{-0.3s}}{(3s-1)(1s-1)}. \tag{33}$$

Liu et al. [8] have already explained the advantage of their method over that of Tan et al. [7]. In this study, the setpoint tracking controller is designed as $k_d = 3, \lambda_c = 1.7\theta = 0.51$. The resulting $C(s) = \left(1.5s^2 + s + 0.5\right) / (0.51s + 1)^2$, according to Liu et al.'s [8] method. The disturbance estimator F, by Liu et al.'s [8] method, is provided as $K_C = 1.7638$, $\tau_I = 1.8679$, $\tau_D = 2.3042$ (i.e. $\lambda_f = 1.7\theta = 0.51$) in the form of PID controller. The third-order controller by Liu et al. [8] for $\lambda_f = 1.5\theta = 0.45$ is given as:

$$F_{3/3}(s) = \frac{32.82s^3 + 439.41s^2 + 232.64s + 129.79}{0.56s^3 + 0.8s^2 + 100s}. \tag{34}$$

The disturbance estimator by the proposed method in the form of PID with a lead–lag filter is given by:

$K_C = 3.5671, \quad \tau_I = 1.491, \quad \tau_D = 1.3364, \quad \alpha = 0.15,$
and $\quad \beta = 0.0058 \quad$ for $\lambda_f = 0.35$.

For performance comparison, a unit step change to the setpoint input at $t = 0$ and an inverse unit step change of load disturbance to the process input at $t = 10$ are added.

In order to evaluate the robustness of each controller, Ms values have been calculated and are given as $Ms = 3.14$, 3.85 and 7.69 for the proposed, Liu et al.'s high order and PID controllers, respectively.

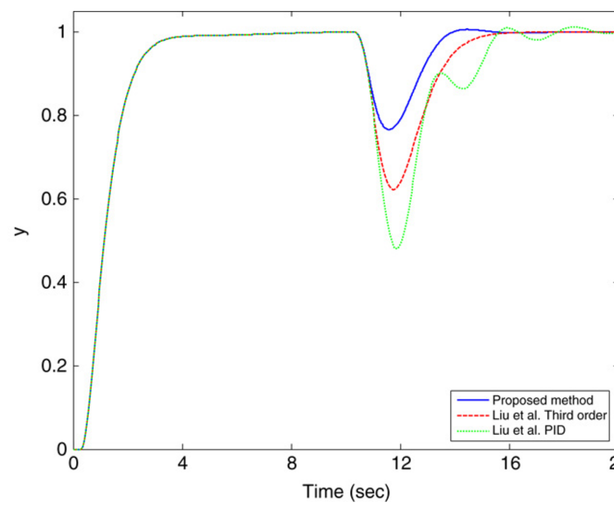

Fig. 2. Response for Example 1 (nominal).

The simulation results are shown in Fig. 2. The proposed disturbance estimator has a faster settling time and smaller peak than either the high order or PID controller by Liu et al. [8]. The PID controller by Liu et al. [8] shows the longest settling time with oscillation. The control effort has been also estimated in terms of TV value. The results are $TV = 4.50, 5.41$ and 9.73 for each controller, consecutively.

Fig. 3 shows the responses for a simultaneous perturbation of 5% in each process parameter towards the worst case model mismatch, which is given as $G_p = 2.1e^{-0.315s} / (2.85s - 1)(0.95s - 1)$. Since Liu et al.'s [8] disturbance estimator in the PID form provides a completely unstable response, it is not shown in the figure. As seen from the figures and their Ms and TV values, the proposed method gives better performance than both the third-order and PID controllers by Liu et al. [8].

Example 2 (*Unstable Process with Large Time Delay*). The following unstable process with comparatively large time delay is studied by Liu et al. [8] and Yang et al. [5].

$$G_p = \frac{e^{-1.2s}}{(1s-1)(0.5s+1)}. \tag{35}$$

The above unstable process is transformed into the form of Eq. (12) and the process parameters for the controller design are obtained as: $k = -1$, $\tau_1 = 1$, $\tau_2 = -0.5$, and $\theta = 1.2$.

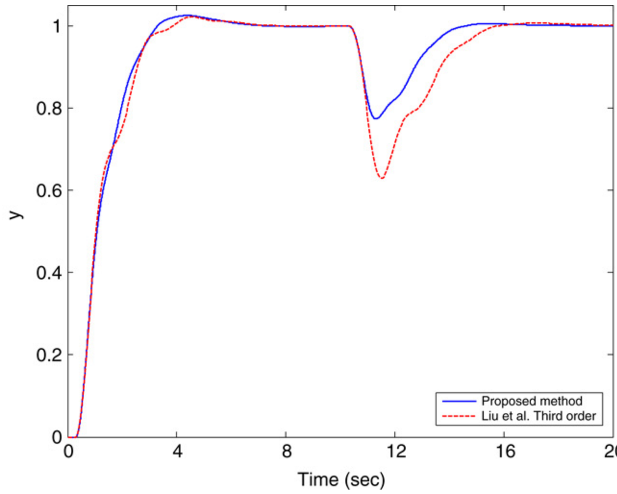

Fig. 3. Response for Example 1 (model mismatch).

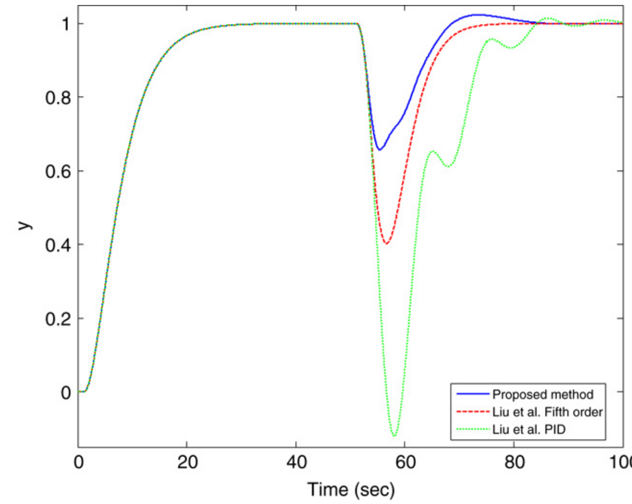

Fig. 4. Response for Example 2 (nominal).

Two previous studies Tan et al. [7] and Majhi and Atherton [10] emphasized the extreme difficulty in using a PID type controller to control the large time delay process mentioned above. As Liu et al. [8] mentioned, the methods of Tan et al. [7] and Majhi and Atherton [10] are unable to damp down a step load disturbance with magnitude of 0.05. The modified IMC–PID approach suggested by Yang et al. [5] shows a severe oscillation. To obtain a smooth and fast response, they proposed a high-order controller numerically derived by using the RLS algorithm. Liu et al. [8] have recently explained the advantage of their method over several other well-known design methods.

The setpoint tracking controller is designed as $k_c = 2$ and $\lambda_c = 3.6$ and $C(s) = \left(0.5s^2 + 0.5s + 1\right) / (3.6s + 1)^2$ according to Liu et al.'s [8] method. The PID parameters of the disturbance estimator by Liu et al.'s [8] method are $K_C = 1.05$, $\tau_I = 245.8901$, $\tau_D = 0.881$ for $\lambda_f = 3.2$ and their fifth-order controller is given below for $\lambda_f = 2.2$.

$$F_{5/5}(s) = \frac{36.12s^5 + 337.86s^4 + 1168.12s^3 + 1331.04s^2 + 120.05s + 1}{s\left(2.08s^4 + 122.32s^3 + 9.84s^2 + 1131.54s + 100\right)}. \quad (36)$$

The controller parameters by the proposed method are $K_C = 1.1165$, $\tau_I = 61.3412$, $\tau_D = 0.4983$, $\alpha = 0.6$, and $\beta = 0.0145$ for $\lambda_f = 1.3$. A unit step change is made to the setpoint input at $t = 0$ and an inverse step change of load disturbance with magnitude of 0.05 to the process input at $t = 50$. Fig. 4 presents the simulation results to compare the proposed controller with Liu et al.'s [8] PID and fifth-order controller. The proposed method clearly shows the significant advantage of load disturbance performance. The disturbance estimator of Liu et al.'s [8] method in the form of PID gives a long peak and significant oscillation, while their fifth-order controller has a smooth response.

The *TV* values have been also calculated as $TV = 0.86$, 1.30 and 2.67 for the proposed, Liu et al.'s fifth-order and PID controllers, respectively.

The robust performance of the controller has been investigated for model mismatch by simultaneously inserting a perturbation uncertainty of 5% in each parameter towards the worst direction and assuming the actual process as $G_p = 1.05e^{-1.26s} / (0.95s - 1)(0.475s + 1)$. Fig. 5 shows the perturbed response of the proposed controller and Liu et al.'s [8] fifth-order controller. Liu et al.'s [8] fifth-order disturbance estimator gave an unstable oscillatory response and their PID controller showed a fully unstable response (data not shown in Fig. 5). The proposed method shows better performance with less control effort in nominal response. For the model mismatch, both the servo and regulatory response was stable and satisfactory.

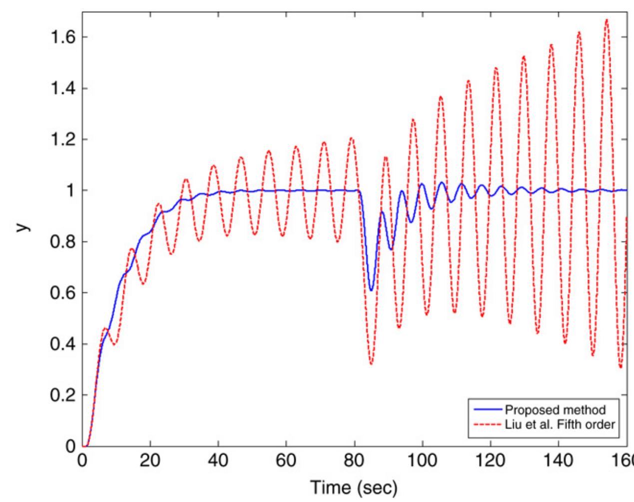

Fig. 5. Response for Example 2 (model mismatch).

Example 3 (*Integrating and Unstable Process*). The unstable process with an integrator, previously studied by Liu et al. [8] and Lee et al. [2], is considered.

$$G_p = \frac{e^{-0.2s}}{s(1s - 1)}. \quad (37)$$

To use the proposed method the above process is modified to the following second-order unstable process:

$$G_p = \frac{100e^{-0.2s}}{(100s - 1)(1s - 1)}. \quad (38)$$

The controller parameters for the proposed disturbance estimator are $K_C = 3.0241$, $\tau_I = 1.7941$, $\tau_D = 1.058$, $\alpha = 0.10$ and $\beta = 0.0087$ for $\lambda_f = 0.40$. The proposed disturbance estimator is compared with Liu et al.'s [8] one with $K_C = 1.4738$, $\tau_I = 2.5712$, $\tau_D = 1.683$ (i.e. $\lambda_c = \lambda_f = 3\theta = 0.6$). For the servo response in both the proposed and Liu et al.'s [8] methods, $k_c = 1$ and $k_d = 2$ are selected to stabilize the setpoint response and $C(s) = \left(s^2 + s + 1\right) / (0.6s + 1)^2$. On the basis of the PID parameters setting above, the *Ms* and *TV* values of

Fig. 6. Response for Example 3 (nominal).

Fig. 7. Response for Example 3 (model mismatch).

both the proposed and Liu et al.'s method have been calculated: the proposed method $Ms = 1.83, TV = 2.96$; the Liu et al. $Ms = 2.33, TV = 3.75$. A unit step change is added to the setpoint input at $t = 0$ and an inverse unit step change of load disturbance is added to the process input at $t = 15$. The simulation results are provided in Fig. 6, which clearly demonstrates the superior enhancement provided by the proposed controller for the regulatory problem with small overshoot and fast settling time, in comparison with Liu et al. [8]. The resulting Ms and TV values also validate superiority of the proposed method.

In the case of a 25% error in estimating the process parameters towards the worst case model mismatch, i.e., $G_p = 1.25e^{-0.25s}/s\,(0.75s - 1)$, the perturbed system responses are shown in Fig. 7. The result demonstrates that the proposed method facilitates better robust stability for both the servo and regulatory problem than does Liu et al.'s [8] method. These results confirm the noteworthy performance improvement of the proposed method for both the nominal case and that with severe process uncertainty.

A comparison has been performed for checking the stability region of the perturbation in the time delay, the time constant, and the process gain by utilizing the Kharitonov's theorem [15]. It states that the Hurwitz stability of the real (or complex) interval polynomial family can be guaranteed by the Hurwitz stability of four prescribed critical vertex polynomials in this family. The number of critical vertex polynomials to be checked is independent of the order of the polynomial family. The result is important since it significantly reduces the stability evaluation for infinitely many polynomials.

Every polynomial in the family $\chi(s)$ is Hurwitz stable if and only if the following four extreme polynomials are Hurwitz stable:

$$k_1(s) = \underline{\chi}_0 + \bar{\chi}_1 s + \bar{\chi}_2 s^2 + \underline{\chi}_3 s^3 + \underline{\chi}_4 s^4 + \bar{\chi}_5 s^5 + \bar{\chi}_6 s^6 \ldots \quad (39a)$$

$$k_2(s) = \underline{\chi}_0 + \underline{\chi}_1 s + \bar{\chi}_2 s^2 + \bar{\chi}_3 s^3 + \underline{\chi}_4 s^4 + \underline{\chi}_5 s^5 + \bar{\chi}_6 s^6 \ldots \quad (39b)$$

$$k_3(s) = \bar{\chi}_0 + \underline{\chi}_1 s + \underline{\chi}_2 s^2 + \bar{\chi}_3 s^3 + \bar{\chi}_4 s^4 + \underline{\chi}_5 s^5 + \underline{\chi}_6 s^6 \ldots \quad (39c)$$

$$k_4(s) = \bar{\chi}_0 + \bar{\chi}_1 s + \underline{\chi}_2 s^2 + \underline{\chi}_3 s^3 + \bar{\chi}_4 s^4 + \bar{\chi}_5 s^5 + \underline{\chi}_6 s^6 \ldots \quad (39d)$$

The stability of the above four equations formed from Kharitonov polynomials is to be checked. For the fixed values of gain k and time constant τ, a perturbation in time delay θ, i.e., $(\theta - \Delta\theta) \le \theta \le (\theta + \Delta\theta)$, is substituted in the above coefficients and Kharitonov's four equations are checked for stability using the Routh–Hurwitz method. Similar perturbation analysis can also be performed for k, i.e., $(k - \Delta k) \le k \le (k + \Delta k)$ (for fixed τ and θ), and τ, i.e., $(\tau - \Delta\tau) \le \tau \le (\tau + \Delta\tau)$ (for fixed k and θ).

The characteristic equation for the closed-loop system is $1 + G_{OL} = 0$. By substitution of G_{OL} and approximating the dead time term by a Pade approximation, the general form of the characteristic equation $1+G_{OL} = 0$ for the integrating and unstable process can be extracted as:

$$\chi(s) = \chi_0 + \chi_1 s + \chi_2 s^2 + \chi_3 s^3 + \chi_4 s^4 + \chi_5 s^5 \quad (40)$$

$\underline{\chi}_i \le \chi_i \le \bar{\chi}_i$, $(i = 0, 1, 2, 3, 4, 5)$ where $\underline{\chi}_i$ and $\bar{\chi}_i$ are the lower and the upper bound for χ_i, respectively.

Consider the control system design of the integrating and unstable process by the PID with a lead–lag filter controller form in Eq. (1). The characteristic equation is given as:

$$1 + \frac{K_c k\left(1 + \tau_I s + \tau_I \tau_D s^2\right)(1 + \alpha s)\, e^{-\theta s}}{\tau_I s^2 (\tau s - 1)(1 + \beta s)} = 0. \quad (41)$$

The time delay term in Eq. (41) has been approximated by using Eq. (16) and the coefficient of the characteristic equation is given below:

$$\chi_0 = K_c k \quad (42a)$$

$$\chi_1 = -0.5 K_c k\theta + K_c k\alpha + K_c k\tau_I \quad (42b)$$

$$\chi_2 = -0.5\theta\,(K_c k\alpha + K_c k\tau_I) + K_c k\tau_I\alpha + K_c k\tau_I\tau_D - \tau_I \quad (42c)$$

$$\chi_3 = -0.5\theta\,(K_c k\tau_I\alpha + K_c k\tau_I\tau_D) + K_c k\tau_I\tau_D\alpha - 0.5\tau_I\theta + \tau_I\tau - \tau_I\beta \quad (42d)$$

$$\chi_4 = 0.5\theta\,(\tau_I\tau - \tau_I\beta) + \tau_I\beta\tau - 0.5K_c k\tau_I\tau_D\alpha\theta \quad (42e)$$

$$\chi_5 = 0.5\tau_I\beta\tau\theta. \quad (42f)$$

Eq. (42) is the coefficient of the characteristic equation in Eq. (40). Uncertainty in the process parameters can be checked for the stability of all four Kharitonov polynomials in Eq. (39).

On the basis of Kharitonov's theorem described above, the parametric uncertainties for the proposed and the Liu et al.'s method has been computed and given as: for the proposed method, $k = \pm 0.48, \tau = \pm 0.545, \theta = \pm 0.186$ and for the Liu et al.'s method $k = \pm 0.39, \tau = \pm 0.535, \theta = \pm 0.17$. The larger uncertainty boundaries for robust stability indicate that the controller designed by the proposed method found more robust than Liu et al. [8].

Remark 2 (*λ Guideline*). In the proposed method, λ_c and λ_f are the user-defined parameters to adjust performance and robustness of the set-point and disturbance rejection, respectively. It is