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Research article

Design of IMC filter for PID control strategy of open-loop unstable processes with time delay

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ABSTRACT: A modified internal model control (IMC) filter has been proposed for the open-loop unstable process with time delay and a further IMC–proportional-integral-derivative (PID) tuning rule is derived on the basis of the proposed IMC filter. Investigation of the IMC filter structure clearly demonstrates that a critically damped filter does not always provide satisfactory response and that this is probably due to lack of sufficient integral action. The conventional, critically damped filter is therefore replaced with a more general, second-order IMC filter for improved integral action. The present investigation shows that for the most unstable processes, the underdamped IMC filter provides the desired integral action and improves the closed-loop performance of the system except for the unstable processes with a strong lead term. An exhaustive simulation studies has been performed to show the advantage of the proposed method in both nominal and robust cases. By incorporating Kharitonov's theorem, closed-loop time constant (λ) guidelines have been provided for the first-order delayed unstable process (FODUP). © 2010 Curtin University of Technology and John Wiley & Sons, Ltd.

KEYWORDS: unstable process; PID controller tuning; dead time process; IMC–PID design; robust analysis; IMC filter

INTRODUCTION

In process control, majority of control loops are proportional-integral-derivative (PID) type, primarily due to its relatively simple structure, which can be easily understood and implemented in practice. It is well-known that control system design for an open-loop unstable process is more difficult than that for a stable one because of the unstable nature of the dynamics, for which most design tools cannot be used. Many of the important chemical processing units in industrial and chemical practice are open-loop unstable processes that are difficult to control, especially in the presence of a time delay, such as in the case of continuous stirred tank reactors, polymerization reactors and bioreactors which are inherently open-loop unstable by design.[1–3]

Furthermore, many of these processes are usually run batchwise, and as a result of possible formulation changes, may operate with significant batch-to-batch variability. Clearly, the tuning of controllers to stabilize these processes and to impart adequate disturbance rejection is critical.

Paor and Malley[4] proposed the tuning method for the first-order delayed unstable process (FODUP). The method is based on the gain and phase margins criterion. Visioli[5] proposed tuning formulas to optimize PID parameters in terms of integral error specifications via a genetic algorithm. However, these conventional PID design methods show excessive overshoots and large settling times. To solve these problems, double-loop schemes have been employed for performance enhancement.[6,7] Huang and Chen[8] suggested a three-element structure for controlling the open-loop unstable processes. However, PID controllers tuned by this method give about 100% overshoot for set-point change. Jung *et al*.[9] suggested a tuning method by using a first-order set-point filter to construct a two-degree-of-freedom (2DOF) control structure for first-order unstable processes.

The effectiveness of the internal model control (IMC) design principle has made this method attractive in the process industry and concentrated efforts on exploiting the IMC principle to design equivalent feedback controllers for unstable processes.[10] The IMC-based

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PID tuning rules have the advantage of using only one tuning parameter to achieve a clear trade-off between closed-loop performance and robustness.

The IMC structure is very powerful for controlling stable processes with time delay, but it cannot be directly used for unstable processes because of the internal instability.[10] For this reason, some modified IMC methods of 2DOF control were developed for controlling unstable processes with time delay.[7,11–14] In addition, 2DOF control methods based on the Smith predictor (SP) were proposed by several authors[15–17] to achieve a smooth nominal set-point response without overshoot for the FODUP. Both the modified IMC and modified SP methods feature the worthy advantage of the nominal set-point response tending to be faster without overshoot for unstable processes. In fact, the common characteristic of these modified IMC and SP methods is the use of a nominal process model in their control structures, which is responsible for their good performance in this respect. Most existing 2DOF control methods are restricted to unstable processes in the form of a first-order rational part plus time delay, which, in fact, cannot sufficiently represent a variety of industrial and chemical unstable processes. Besides, there usually exist unmodeled dynamics that inevitably tend to deteriorate the control system performance.

Zhou *et al*.[18] have conducted a comparative study for understanding the time-delayed effect on unstable processes. On the bases of performance and robustness, Zhou *et al*.[18] suggested that the tuning methods presented by Lee *et al*.[11] and Yang *et al*.[12] are applicable in large normalized dead time $\theta/\geq\tau$. Xiang and Nguyen[19] have suggested a control design method for the FODUP with a complicated three-element control structure. This method is advantageous in the set-point performance but in the disturbance rejection it shows big overshoot and long settling time. The controller design for second-order delayed unstable process (SODUP) systems with two unstable poles and a negative zero has been described by Lee *et al*.[11] and Wang and Hwang[20]. However, the controller design for SODUP systems with one/two right-half-plane (RHP) poles and an RHP zero has not yet been addressed properly.

It is apparent from the literature that only few tuning methods are available for dead time dominant unstable processes and the processes which contain one/two unstable poles and positive and negative zeros. The above discussion demonstrates that a tuning method is required to cover several classes of the process model, including positive and negative zeros, and also various dead time dominant unstable processes.

In this study, the dead time dominant unstable process with positive and negative zeros was investigated. The modified IMC filter has been suggested and categorized for several classes of the process for improved disturbance rejection. Excessive overshoot in the set-point response is eliminated by a set-point filter. On the basis of extensive simulation study, closed-loop time constant (λ) guidelines are proposed for 5% and 10% dead time uncertainty by using Kharitonov's theorem[21] at different damping coefficient (ζ) values. The simulation studies confirm the advantage of the proposed method over other recently published methods for several representative classes of the process model while maintaining the same robustness according to the measure of maximum sensitivity, Ms.

IMC–PID CONTROLLER DESIGN PROCEDURE

Figure 1(a) presents a block diagram of the IMC control structure, where G_p is the process, $\tilde{G}_p$ the process model, and q the IMC controller. In the IMC control structure, the controlled variable is related as follows:

$$C = \frac{G_p q}{1 + q\left(G_p - \tilde{G}_p\right)} f_R R + \left[\frac{1 - \tilde{G}_p q}{1 + q\left(G_p - \tilde{G}_p\right)}\right] G_p d \tag{1}$$

For the nominal case (i.e. $G_p = \tilde{G}_p$), the set-point and disturbance responses are simplified as below:

$$\frac{C}{R} = G_p q f_R \tag{2}$$

$$\frac{C}{d} = (1 - G_p q) G_p \tag{3}$$

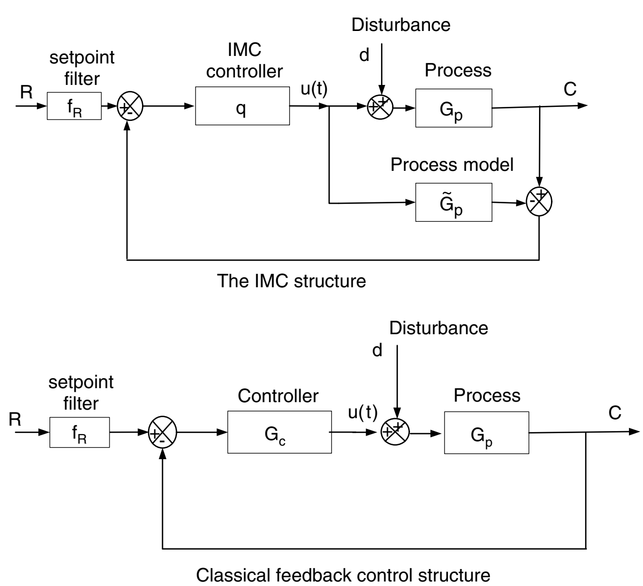

Figure 1. Block diagram of IMC and classical feedback control. (a) The IMC structure. (b) Classical feedback control structure.

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In the classical feedback control structure shown in Fig. 1(b), the set-point and disturbance responses are represented by

$$\frac{C}{R} = \frac{G_c G_p f_R}{1 + G_c G_p} \tag{4}$$

$$\frac{C}{d} = \frac{G_p}{1 + G_c G_p} \tag{5}$$

where G_c denotes the feedback controller. The IMC controller is a competent method for control system design.[10] However, for unstable processes, the IMC structure cannot be implemented because any bounded input, d, will produce unbound output, C, if G_p is unstable. As discussed by Morari and Zafiriou,[10] the IMC approach to designing a controller for an unstable process is possible for $G_p = \tilde{G}_p$ if the following conditions are satisfied for the internal stability of the closed-loop system:

1. q is stable.
2. $G_p q$ is stable.
3. $(1 - G_p q)G_p$ is stable.

These three conditions result in the well-known, standard interpolation conditions:[10]

If the process model, G_p, has unstable poles, up_1, $up_2, \cdots, up_m$, then q should have zeros at $up_1, up_2, \cdots$, up_m and $1 - G_p q$ should have zeros at $up_1, up_2, \cdots$, up_m.

According to the IMC parameterization, the process model $\tilde{G}_p$ is decomposed into two parts:

$$\tilde{G}_p = P_M P_A \tag{6}$$

where P_M and P_A are the portions of the model inverted and not inverted, respectively, by the controller (P_A is usually a nonminimum phase and contains dead times and/or RHP zeros); $P_A(0) = 1$.

Since the IMC controller, q, is designed as $q = P_M^{-1} f$ in which the IMC filter, f, is stable and P_M^{-1} includes the inverse of the unstable portion, the controller satisfies the first two conditions. The third condition could be satisfied through the design of the IMC filter, f. For this, the filter is designed as

$$f = \frac{\sum_{i=1}^{m} \beta_i s^i + 1}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} \tag{7}$$

where m is the number of poles to be canceled, β_i is determined by Eqn (8) to cancel the unstable poles in G_p, and r is selected as being large enough to make the IMC controller proper.

$$1 - G_p q\,|_{s=up_1, up_2, \cdots, up_m} = \left| 1 - \frac{P_A(\sum_{i=1}^{m} \beta_i s^i + 1)}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} \right|_{s=up_1, up_2, \cdots, up_m} = 0 \tag{8}$$

Then, the IMC controller is

$$q = P_M^{-1} \frac{\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} \tag{9}$$

Thus, the resulting set-point and disturbance responses are obtained as

$$\frac{C}{R} = G_p q f_R = P_A \frac{\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} f_R \tag{10}$$

$$\frac{C}{d} = \left(1 - G_p q\right) G_p = \left(1 - P_A \frac{\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r} \right) G_p \tag{11}$$

The expression, $\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)$, in the numerator in Eqn (10) causes an unreasonable overshoot in the servo response, which can be eliminated by designing the set-point filter f_R as

$$f_R = \frac{1}{\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)} \tag{12}$$

The resulting IMC controller in Eqn (9) has a stable response. A classical feedback controller equivalent to the IMC controller can be obtained from the following relationship:

$$G_c = \frac{q}{1 - \tilde{G}_p q} \tag{13}$$

Thus the feedback controller is given as

$$G_c = \frac{P_M^{-1} \left(\sum_{i=1}^{m} \beta_i s^i + 1\right)}{(\lambda^2 s^2 + 2\lambda\zeta s + 1)^r - P_A \left(\sum_{i=1}^{m} \beta_i s^i + 1\right)} \tag{14}$$

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The resulting feedback controller given by Eqn (14) is physically realizable, but it lacks the standard PID controller form. Therefore, the next step is to design the PID controller that most closely approximates the equivalent feedback controller. The Maclaurin series approximation has been utilized to approximate Eqn (14) to the PID controller.

Lee *et al*.[22] proposed an efficient method for converting the ideal feedback controller G_c to a standard PID controller. Since G_c has an integral term, it can be expressed as

$$G_c = \frac{g(s)}{s} \tag{15}$$

Expanding G_c in the Maclaurin series in s gives

$$G_c = \frac{1}{s}\left(g(0) + g'(0)s + \frac{g''(0)}{2}s^2 + \ldots\right) \tag{16}$$

The first three terms of the above expansion can be interpreted as the standard PID controller, which is given by

$$G_c = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \tag{17}$$

where

$$K_c = g'(0) \tag{18a}$$

$$\tau_I = g'(0)/g(0) \tag{18b}$$

$$\tau_D = g''(0)/2g'(0) \tag{18c}$$

Design method for proposed tuning rule

Consider a FODUP as a representative model of the form

$$G_p = \frac{Ke^{-\theta s}}{\tau s - 1} \tag{19}$$

where K is the gain, τ the time constant, and θ the time delay. The proposed IMC filter is found as $f = (\beta s + 1)/(\lambda^2 s^2 + 2\lambda\zeta s + 1)$. The resulting IMC controller becomes $q = (\tau s - 1)(\beta s + 1)/K(\lambda^2 s^2 + 2\lambda\zeta s + 1)$. Therefore, the ideal feedback controller equivalent to the IMC controller is

$$G_c = \frac{(\tau s - 1)(\beta s + 1)}{K\left[(\lambda^2 s^2 + 2\lambda\zeta s + 1) - e^{-\theta s}(\beta s + 1)\right]} \tag{20}$$

The analytical PID formula can be obtained from Eqn (18) as

$$K_c = \frac{\tau_I}{K(\theta - \beta + 2\lambda\zeta)} \tag{21}$$

$$\tau_I = (-\tau + \beta) - \frac{\left(\lambda^2 - \theta^2/2 + \beta\theta\right)}{(\theta - \beta + 2\lambda\zeta)} \tag{22}$$

$$\tau_D = \frac{(-\tau\beta) - \dfrac{\left(\theta^3/6 - \beta\theta^2/2\right)}{(\theta - \beta + 2\lambda\zeta)}}{\tau_I} - \frac{\left(\lambda^2 - \theta^2/2 + \beta\theta\right)}{(\theta - \beta + 2\lambda\zeta)} \tag{23}$$

The extra DOF β is calculated by solving $(1 - Gq)|_{s=1/\tau} = 0$. This means β is chosen so that the term $(1 - Gq) = 0$ at the pole of G_p, which leaves $\left[1 - (\beta s + 1)e^{-\theta s}/(\lambda^2 s^2 + 2\lambda\zeta s + 1)\right]\Big|_{s=1/\tau} = 0$. The value of β after some simplification is

$$\beta = \tau\left[(\lambda^2 + 2\lambda\zeta\tau + \tau^2)e^{\theta/\tau}/\tau^2 - 1\right] \tag{24}$$

On the basis of the above design principle, the PID tuning formulas for several process models are obtained and listed in Table 1.

Remark 1. The design of the IMC–PID controller based on the proposed method is not applicable to the second-order process with complex conjugate pairs. It is due to the value of β_1 and β_2 comes out complex conjugate pairs during the calculation of the PID parameters, which gives unrealistic PID controller settings.

Remark 2. The processes E1, E2, E3, and E4 may have negative zero in the process transfer function in the form of $(\tau_a s + 1)$. For illustration purposes, the second-order unstable process with negative zero is given as $G_p = (\tau_a s + 1)Ke^{-\theta s}/(\tau_1 s - 1)(\tau_2 s - 1)$. An additional filter of the form $f_a = 1/(\tau_a s + 1)$ with PID controller is obtained for the processes with the negative zero and the resulting PID controller should be in the form $G_c = K_c(1 + 1/\tau_I s + \tau_D s).f_a$.

In the process with the positive zero or inverse response, the negative numerator time constant is approximated as a time delay $(-\tau_a s + 1 \approx e^{-\tau_a s})$. This is reasonable since an inverse response has a deteriorating effect on control, similar to that of a time delay.[23] The proposed method is applicable for any positive zero after approximation of the negative numerator time constant as an effective time delay, e.g. $G_p = (-\tau_a + 1)e^{-\theta s}/(\tau_1 s - 1)(\tau_2 s + 1) = e^{-(\theta+\tau_a)s}/(\tau_1 s - 1)(\tau_2 s + 1)$ and this is illustrated in Examples 5 and 6.

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Table 1. IMC–PID controller-tuning rules for unstable processes.

Case	Process model	K_c	τ_I	τ_D	β
E1	$\dfrac{K e^{-\theta s}}{\tau s - 1}$	$-\dfrac{\tau_I}{K(\theta - \beta + 2\lambda\zeta)}$	$(\beta - \tau) - \dfrac{\left(\lambda^2 - \dfrac{\theta^2}{2} + \theta\beta\right)}{(\theta - \beta + 2\lambda\zeta)}$	$\dfrac{-\tau\beta - \dfrac{\left(\dfrac{\theta^3}{6} - \dfrac{\beta\theta^2}{2}\right)}{(\theta - \beta + 2\lambda\zeta)}}{\tau_I} - \dfrac{\left(\lambda^2 - \dfrac{\theta^2}{2} + \theta\beta\right)}{(\theta - \beta + 2\lambda\zeta)}$	$\beta = \tau\left[\dfrac{(\lambda^2 + 2\lambda\zeta\tau + \tau^2)e^{\theta/\tau}}{\tau^2} - 1\right]$
E2	$\dfrac{K e^{-\theta s}}{(\tau_1 s - 1)(\tau_2 s + 1)}$	$-\dfrac{\tau_I}{K(\theta - \beta + 2\lambda\zeta)}$	$(\beta - \tau_1 + \tau_2) - \dfrac{\left(\lambda^2 - \dfrac{\theta^2}{2} + \theta\beta\right)}{(\theta - \beta + 2\lambda\zeta)}$	$\dfrac{(\tau_2 - \tau_1)\beta - \tau_1\tau_2 - \dfrac{\left(\dfrac{\theta^3}{6} - \dfrac{\beta\theta^2}{2}\right)}{(\theta - \beta + 2\lambda\zeta)}}{\tau_I} - \dfrac{\left(\lambda^2 - \dfrac{\theta^2}{2} + \theta\beta\right)}{(\theta - \beta + 2\lambda\zeta)}$	$\beta = \tau_1\left[\dfrac{(\lambda^2 + 2\lambda\zeta\tau_1 + \tau_1^2)e^{\theta/\tau_1}}{\tau_1^2} - 1\right]$

Table 1. (*Continued*).

Case	Process model	K_c	τ_I	τ_D	β
E3	$\dfrac{K\mathrm{e}^{-\theta s}}{(\tau_1 s-1)(\tau_2 s-1)}$	$\dfrac{\tau_I}{K(\theta-\beta_1+4\lambda\zeta)}$	$(\beta_1-\tau_1-\tau_2)-\dfrac{\left(-\dfrac{\theta^2}{2}+\theta\beta_1-\beta_2+2\lambda^2+4\lambda^2\zeta^2\right)}{(\theta-\beta_1+4\lambda\zeta)}$	$\dfrac{\beta_2-(\tau_1+\tau_2)\beta_1+\tau_1\tau_2-\dfrac{\left(\dfrac{\theta^3}{6}-\dfrac{\beta_1\theta^2}{2}+\theta\beta_2+4\lambda^3\zeta\right)}{(\theta-\beta_1+4\lambda\zeta)}}{\tau_I}-\dfrac{\left(-\dfrac{\theta^2}{2}+\theta\beta_1-\beta_2+2\lambda^2+4\lambda^2\zeta^2\right)}{(\theta-\beta_1+4\lambda\zeta)}$	$\beta_1=\dfrac{\tau_1{}^2\left(\dfrac{\lambda^2+2\lambda\zeta\tau_1+\tau_1{}^2}{\tau_1{}^2}\right)^2\mathrm{e}^{\theta/\tau_1}-\tau_2{}^2\left(\dfrac{\lambda^2+2\lambda\zeta\tau_2+\tau_2{}^2}{\tau_2{}^2}\right)^2\mathrm{e}^{\theta/\tau_1}-\left(\tau_1{}^2-\tau_2{}^2\right)}{(\tau_1-\tau_2)}$ $\beta_2=\tau_1{}^2\left(\dfrac{\lambda^2+2\lambda\zeta\tau_1+\tau_1{}^2}{\tau_1{}^2}\right)^2 e^{\theta/\tau_1}-\left(1+\dfrac{\beta_1}{\tau_1}\right)$
E4	$\dfrac{K\mathrm{e}^{-\theta s}}{s(\tau_1 s-1)}$ $=\dfrac{K\psi\mathrm{e}^{-\theta s}}{(\psi s-1)(\tau_1 s-1)}$	$\dfrac{\tau_I}{K(\theta-\beta_1+4\lambda\zeta)}$	$(\beta_1-\tau_1-\psi)-\dfrac{\left(-\dfrac{\theta^2}{2}+\theta\beta_1-\beta_2+2\lambda^2+4\lambda^2\zeta^2\right)}{(\theta-\beta_1+4\lambda\zeta)}$	$\dfrac{\beta_2-(\tau_1+\psi)\beta_1+\tau_1\psi-\dfrac{\left(\dfrac{\theta^3}{6}-\dfrac{\beta_1\theta^2}{2}+\theta\beta_2+4\lambda^3\zeta\right)}{(\theta-\beta_1+4\lambda\zeta)}}{\tau_I}-\dfrac{\left(-\dfrac{\theta^2}{2}+\theta\beta_1-\beta_2+2\lambda^2+4\lambda^2\zeta^2\right)}{(\theta-\beta_1+4\lambda\zeta)}$	$\beta_1=\dfrac{\tau_1{}^2\left(\dfrac{\lambda^2+2\lambda\zeta\tau_1+\tau_1{}^2}{\tau_1{}^2}\right)^2\mathrm{e}^{\theta/\tau_1}-\psi^2\left(\dfrac{\lambda^2+2\lambda\zeta\psi+\psi^2}{\psi^2}\right)^2\mathrm{e}^{\theta/\psi}-\left(\tau_1{}^2-\psi^2\right)}{(\tau_1-\psi)}$ $\beta_2=\tau_1{}^2\left(\dfrac{\lambda^2+2\lambda\zeta\tau_1+\tau_1{}^2}{\tau_1{}^2}\right)^2 e^{\theta/\tau_1}-\left(1+\dfrac{\beta_1}{\tau_1}\right)$

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SIMULATION STUDY

This section demonstrates the simulation study of several representative unstable processes with dead time, including positive and negative zeros.

To evaluate the closed-loop performance, the integral of the time-weighted absolute error (ITAE) criterion was considered in the case of both a step set-point change and a step load disturbance. The ITAE is defined as

$$\mathrm{ITAE} = \int_0^\infty t|\mathrm{e}(t)|\mathrm{d}t \qquad (25)$$

For the evaluation of the smoothness of a signal, the total variation (TV) of the input $u(t)$, $\mathrm{TV} = \sum_{i=1}^{\infty} |u_{i+1} - u_i|$, is computed. Overshoot, which is a measure of how much the response exceeds the ultimate value following a step change in the set-point and/or disturbance, has been also calculated for each simulation example.

The maximum sensitivity Ms[23], which is defined as $\mathrm{Ms} = \max|1/[1 + G_p G_c(i\omega)]|$, was used to evaluate the robustness of the control system. Since Ms is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point $(-1, 0)$, a small Ms value indicates that the control system has a large stability margin. To ensure a fair comparison, it is widely accepted that the model-based controllers (IMC and direct synthesis) need to be tuned by adjusting λ to give the same Ms values.

Throughout all the simulation examples, all the controllers compared were designed to have the same robustness level in terms of Ms and the performance indices such as ITAE, overshoot, and TV values are compared.

In this article, the simulation study has been conducted using the PID controller in the form of Eqn (17). However, for real implementation, the "parallel form" of the PID controller, $G_c = K_c(1 + 1/\tau_I s + \tau_D s/0.1\tau_D s + 1)$, which is widely used in the real processes, can be applied to approximately the same performance. The other form of the PID controller can easily be converted from 'parallel form'.[24]

Example 1 Lag time dominant FODUP. An extensively published FODUP model[8,11,13,14,25] was considered for a comparison of the performance:

$$G_p = \frac{\mathrm{e}^{-0.4s}}{(s-1)} \qquad (26)$$

For the above FODUP model, the recently published paper of Liu *et al*.[14] demonstrated the improvement of their method over those of Tan *et al*.[13] and Majhi and Atherton.[25] In this simulation study, the proposed method is compared with those of Liu *et al*.[14] and Lee *et al*.[11] The design of the disturbance rejection is identical for the methods of both Liu *et al*.[14] and Lee *et al*.[11] However, for the set-point response, Liu *et al*.[14] used a modified IMC structure, while Lee *et al*.[11] applied a set-point filter. For the methods of both Liu *et al*.[14] and Lee *et al*.,[11] $\lambda = 0.4$ was used in the simulation, producing Ms = 3.65. To obtain a fair comparison, λ is also adjusted in the proposed method ($\lambda = 0.401$) to obtain Ms = 3.65. The controller parameters, including the performance and robustness matrix, are listed in Table 2.

Figure 2 presents a comparison of the proposed method with other methods by introducing a unit step change in the set-point at $t = 0$ and an inverse unit step change of load disturbance at $t = 4$. For the servo response, the set-point filter is used for both the proposed method and that of Lee *et al*.[11] whereas a three-controller element structure is used for the method of Liu *et al*.[14] As it is apparent from Fig. 2 and Table 2, the proposed method improves the load disturbance response. Since the design of the disturbance rejection is identical for the methods of Liu *et al*.[14] and Lee *et al*.,[11] the same PID tuning setting and consequently an identical disturbance rejection response is obtained in both cases. For the servo response, the method of Liu *et al*.[14] seems to be better, but the settling times in the method of Liu *et al*.[14] and the proposed method are comparable, while the method of Lee *et al*.[11] shows the slowest response with the longest settling time.

The well-known, modified IMC structure has the theoretical advantage of eliminating the time delay from the characteristic equation. Unfortunately, this advantage is lost if the process model is inaccurate. Besides, real process plants usually incorporate unmodeled dynamics that inevitably tend to deteriorate the control system performance severely. The robustness of the controller was investigated by inserting a perturbation uncertainty of 5% in all three parameters simultaneously toward the worst case model mismatch, i.e. $G_p = 1.05\mathrm{e}^{-0.42s} \big/ (0.95s - 1)$.

The simulation results for model mismatch are also presented in Table 2 for both the set-point and the disturbance rejection. The table indicates that the proposed controller-tuning method has an almost similar load response, while the modified IMC controller corresponding to method of Liu *et al*.'s[14] has the better set-point response for model mismatch and the method of Lee *et al*.[11] shows the highest ITAE value.

Example 2. Dead time dominant FODUP. Consider the following dead time dominant ($\theta\big/\tau = 1.5$) unstable process:[18,19]

$$G_p = \frac{\mathrm{e}^{-1.5s}}{(s-1)} \qquad (27)$$

Recently, Zhou *et al*.[18] conducted a comparative study to show the applicability of several existing controller design methods and evaluated their control performance and robustness and concluded that for the large

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Table 2. Controller parameters and resulting performance indices for Example 1, $G_p = e^{-0.4s}/(s-1)$.

Method	K_c	τ_I	τ_D	Set-point Nominal ITAE	Set-point Nominal Overshoot	Set-point Nominal TV	Set-point 5% Mismatch ITAE	Set-point 5% Mismatch Overshoot	Disturbance Nominal ITAE	Disturbance Nominal Overshoot	Disturbance Nominal TV	Disturbance 5% Mismatch ITAE	Disturbance 5% Mismatch Overshoot
Proposed $\lambda = 0.4$[a]	2.857	1.759	0.152	0.616	1.01	2.63	1.018	1.057	0.791	0.678	4.120	1.714	0.782
Liu *et al.*[14] $\lambda = 0.4$[b]	2.897	2.097	0.161	0.40	1.0	3.50	0.819	1.03	1.008	0.665	4.060	1.771	0.777
Lee *et al.*[11] $\lambda = 0.4$[c]	2.894	2.101	0.161	0.894	1.01	2.30	1.237	1.018	1.012	0.666	4.048	1.756	0.770

Ms = 3.65.
[a] $f_R = 1/(1.5932s + 1)$, $\zeta = 0.72$.
[b] $K_c = 2$, $C(s) = (s + 1)/(0.4s + 1)$.
[c] $f_R = 1/(1.9282s + 1)$.

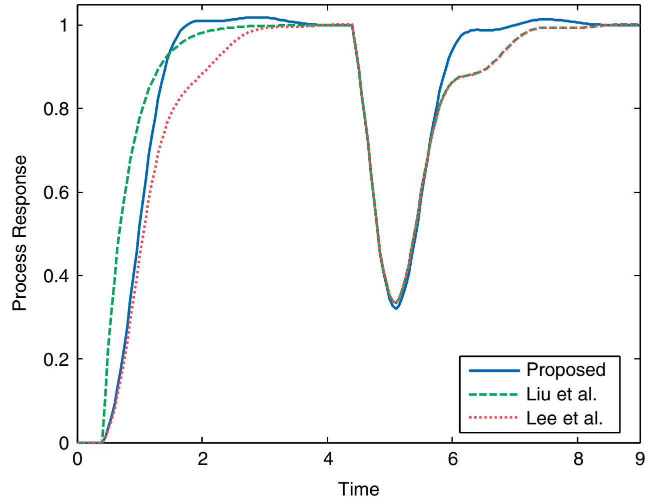

Figure 2. Response of the nominal system for Example 1. This figure is available in colour online at www.apjChemEng.com.

dead time ($\theta/\tau = 1.5$) process, only the methods of Lee *et al.*[11] and Yang *et al.*[12] are applicable among several other tuning methods.[5–7,15,25] Although Zhou *et al.*[18] argued that the methods of both Lee *et al.*[11] and Yang *et al.*[12] are different, both methods have an identical PID controller setting and thus the same performance for the same Ms value. For the performance comparison, the proposed method is compared with the method of Lee *et al.*[11] and the recently published Xiang and Nguyen[19] method. The controller setting parameters are listed in Table 3. To test the performance of the control system, a set-point with a magnitude of 1.0 is added at $t = 0$ and a load disturbance with a step change of magnitude 0.1 at $t = 50$. The simulation results are presented in Fig. 3 and the performance indices in Table 3, where the proposed method produces improved performance in both disturbance rejection and set-point tracking. The servo response of the methods of Lee *et al.*[11] and Xiang and Nguyen[19] is slow, whereas that of the proposed method is fast with little overshoot. Although the Xiang and Nguyen[19] method used a modified control structure comprising three individual controllers for a smooth set-point response, the disturbance rejection shows a very high peak and long settling time.

The robustness of the controller is evaluated by simultaneously inserting a perturbation uncertainty of 1% in all three parameters to obtain the worst case model mismatch, i.e. $G_p = 1.01e^{-1.515s}/(0.99s - 1)$, as an actual process, whereas the controller settings are those calculated based on the nominal model. In Table 3, the performance matrix is also listed both for the set-point and disturbance responses in the model mismatch case. The controller setting in the Xiang and Nguyen[19] method provides robust performance for

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Table 3. Controller parameters and resulting performance indices for Example 2, $G_p = e^{-1.5s}/(s-1)$.

Method	K_c	τ_I	τ_D	Set-point: Nominal ITAE	Set-point: Nominal Overshoot	Set-point: Nominal TV	Set-point: 1.0% Mismatch ITAE	Set-point: 1.0% Mismatch Overshoot	Disturbance: Nominal ITAE	Disturbance: Nominal Overshoot	Disturbance: Nominal TV	Disturbance: 1.0% Mismatch ITAE	Disturbance: 1.0% Mismatch Overshoot
Proposed $\lambda = 4.308$[a]	1.065	106.72	0.757	63.49	1.12	1.353	92.06	1.083	140.90	1.506	4.711	464.6	1.60
Lee *et al.*[11]/Yang *et al.*[12] $\lambda = 6.543$[b]	1.064	254.75	0.763	155.30	1.0	1.013	238.0	1.0	366.20	1.509	4.697	600.0	1.61
Xiang and Nguyen[19]c	0.008	1.4815	9.05	76.82	1.06	1.257	84.84	1.020	916.20	3.153	11.538	497.0	3.21

Ms = 29.70.
[a] $f_R = 1\big/(105.9639s + 1)$, $\zeta = 0.5$.
[b] $f_R = 1\big/(253.994s + 1)$.
[c] $C_2(s) = 1.019 + 0.59s$, $F(s) = 1\big/(5s + 1)$.

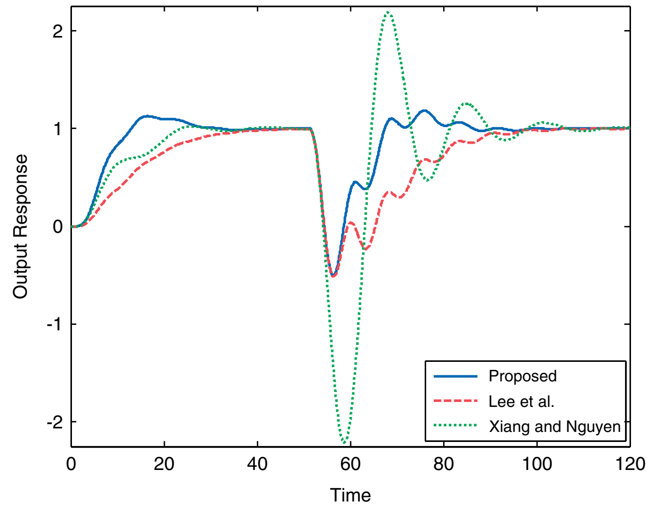

Figure 3. Response of the nominal system for Example 2. This figure is available in colour online at www.apjChemEng.com.

the servo problem but a big overshoot in disturbance rejection. The proposed method and that of Lee *et al.*[11] have almost similar performance in both the servo and regulatory problems.

Example 3. Second-order delayed unstable process (SODUP). The following unstable process is considered for the present study:[8,11–13]

$$G_p = \frac{e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)} \qquad (28)$$

The model presented above is a high-order unstable process and is approximated with an unstable pole with dead time.[8,11–13]

$$G_p = \frac{e^{-0.939s}}{(5s-1)(2.07s+1)} \qquad (29)$$

Tan *et al.*[13] have already explained the advantage of their method over those of Huang and Chen,[8] and Lee *et al.*[11] For this reason, the proposed method is compared with the methods of Tan *et al.*[13] and Yang *et al.*[12] to show its improvement over other methods. The λ value for the proposed method is adjusted to obtain the same Ms = 2.21 as the method of Yang *et al.*[12] and the controller setting parameters with performance indices are listed in Table 4.

For the simulation of the above process, both the set-point and load disturbance that has a step change of magnitude 1 and −1 are inserted at $t = 0$ and at $t = 30$, respectively. The output response for both the servo and regulatory problem is shown in Fig. 4. The proposed tuning method has a faster settling time than that of the other existing methods. Tan *et al.*'s method,[13]

Table 4. Controller parameters and resulting performance indices for Example 3, $G_p = e^{-0.5s}/(5s-1)(2s+1)(0.5s+1) \approx e^{-0.939s}/(5s-1)(2.07s+1)$.

				Set-point					Disturbance				
				Nominal			10% Mismatch		Nominal			10% Mismatch	
Method	K_c	τ_I	τ_D	ITAE	Overshoot	TV	ITAE	Overshoot	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed $\lambda = 2.352$[a]	4.108	8.705	1.750	14.20	1.023	3.044	14.40	1.0	15.37	0.322	2.725	15.04	0.321
Yang *et al.*[12] $\lambda = 1.5$[b]	2.989	12.799	1.562	19.40	1.020	2.345	24.91	1.0	36.47	0.505	2.928	38.12	0.495
Tan *et al.*[13]c	–	–	–	39.94	1.0	5.999	45.60	1.0	26.79	0.275	2.471	27.58	0.270

Ms = 2.21.
[a] $f_R = 1\Big/(105.9639s+1)$, $\zeta = 0.71$.
[b] $f_R = (3.1230s+1)\Big/(19.9858s^2+12.7991s+1)$.
[c] $K_0 = 2(2.07s+1)$, $K_1 = s+1/0.2s+1$, $K_2 = 3.58(2.4s+1)$.

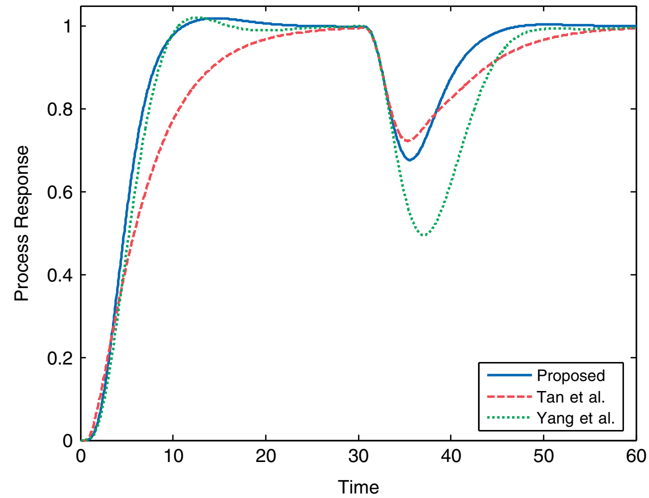

Figure 4. Response of the nominal system for Example 3. This figure is available in colour online at www.apjChemEng.com.

particularly, has a slow response for both the servo and the regulatory problems.

Table 4 presents the performance index as well, with 10% uncertainties in gain and dead time, and a time constant $G_p = 1.1e^{-0.55s}\big/(4.5s-1)(1.8s+1)(0.45s+1)$ toward the worst case model mismatch. The proposed method clearly shows better response for both the set-point tracking and disturbance rejection for the nominal as well as the model mismatch case.

Example 4. Consider the following unstable process with a strong lead-time constant[11,26] and two unstable poles:

$$G_p = \frac{2(5s+1)e^{-0.3s}}{(3s-1)(1s-1)} \tag{30}$$

The methods of Lee *et al.*[11], Rames[26] and the proposed study were used to design the PID controller. For both the proposed method and the method of Lee *et al.*, λ is adjusted to obtain Ms = 4.5 and the controller setting parameters are listed in Table 5. The reported value of the Rames method[26] has been also used for comparison with the proposed method. Figure 5(a) shows the closed-loop output response for a unit-step, set-point change occurring at $t = 0$, and an inverse unit step change of load at $t = 10$. Figure 5(a) demonstrates that the proposed controller provides excellent enhancement for both the servo and regulatory problems over the other methods. For the servo response, the proposed study utilizes a single DOF controller, whereas the method of Lee *et al.*[11] employs a set-point filter to reduce the overshoot in the set-point response. For the disturbance rejection, both overshoot and undershoot are significantly small in the proposed method whereas the method of Lee *et al.* method shows an aggressive undershoot, which is undesirable in real control practices. The Rames method[26] shows quite a

Table 5. Controller parameters and resulting performance indices for Example 4, $G_p = 2(5s+1)e^{-0.3s}/(3s-1)(s-1)$.

Method	K_c	τ_I	τ_D	Set-point Nominal			Set-point 3% Mismatch		Disturbance Nominal			Disturbance 3% Mismatch	
				ITAE	*overshoot*	*TV*	*ITAE*	*overshoot*	*ITAE*	*overshoot*	*TV*	*ITAE*	*overshoot*
Proposed $\lambda = 0.657$[a]	0.8166	2.8178	4.7182	3.71	1.027	1.002	4.880	1.055	14.99	3.3905	6.3218	19.69	3.6657
Lee *et al.*[11] $\lambda = 0.692$[b]	0.7959	1.8973	4.3397	6.065	1.0058	0.764	7.082	1.0292	18.68	3.7391	6.6977	18.73	3.9924
Rames C. Panda[26]c	0.881	5.110	12.1	125.3	1.1524	0.9865	No stable response		92.15	2.6808	18.7564	Not stable response	

For both the proposed and Lee *et al.*[11] methods Ms = 4.5.
[a] No set-point filter is used, $\zeta = 1.6$ and $f_a = 1/(5s+1)$.
[b] $f_R = 1/(8.187s^2 + 1.876s + 1)$ and $f_a = 1/(5s+1)$.
[c] No set-point filter and PID controller in the form of $G_c = K_c(1 + 1/\tau_I s + \tau_D s) \times 1/(10.05s+1)$ and resulting Ms = 10.63.

slow response for the set-point tracking and oscillatory behavior in the disturbance rejection and the resulting Ms value is also high.

The robust performance is evaluated by simultaneously inserting a perturbation uncertainty of 3% in all three parameters in the worst direction and finding the actual process as $G_p = 2.06(5.15s+1)e^{-0.309s}/(2.91s-1)(0.97s-1)$. The simulation results for the model mismatch are also given in Table 5 where the Rames method[26] is unable to provide the stable closed-loop response. The robustness indices demonstrate that the proposed method has more robust performance in terms of Integral of Absolute Error (IAE) and Integral of Squared Error (ISE) for the disturbance rejection, although it is not listed, while the ITAE value is slightly more in the proposed study because of the sluggish response.

The effect of measurement noise on the model parameters is evaluated by adding white noise with noise power 0.001 and sample time 0.1 s to the process output. The process output and control action are shown in Fig. 5(b) and (c). It is clear that robust control performance can be obtained against measurement noise in the system.

Example 5. SODUP with one stable pole and a positive zero. Consider the following SODUP with one stable pole, which was studied in Example 3:

$$G_p = \frac{e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)} \approx \frac{e^{-0.939s}}{(5s-1)(2.07s+1)} \quad (31)$$

The above process is modified for this section with a positive zero as given below:

$$G_p = \frac{(-s+1)e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)} = \frac{(-s+1)e^{-0.939s}}{(5s-1)(2.07s+1)} \quad (32)$$

It is difficult to directly design the PID controller for unstable processes with a positive zero. For the controller design, the above process can be approximated as

$$G_p = \frac{e^{-1.939s}}{(5s-1)(2.07s+1)} \quad (33)$$

The positive zero term is approximated as a time delay ($-\tau_a s + 1 \approx e^{-\tau_a s}$). This is reasonable since an inverse response has a deteriorating effect on control similar to that of a time delay.

For the proposed method, Lee *et al.*[11] and Rao and Chidambaram[2], λ is adjusted to obtain Ms = 3.5. The simulation is conducted for $G_p = (-s+1)e^{-0.5s}/(5s-1)(2s+1)(0.5s+1)$ and the resulting performance

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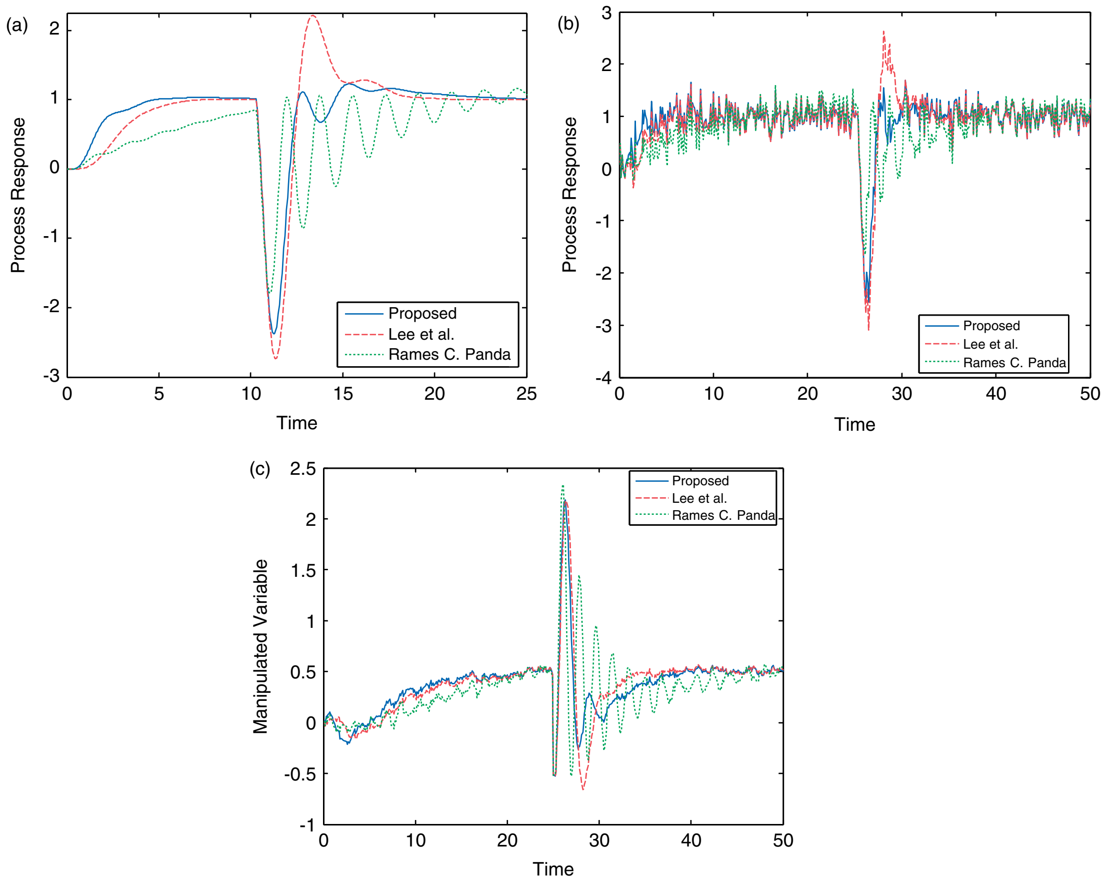

Figure 5. (a)Response of the nominal system for Example 4. (b) Response of the nominal system with white noise of power 0.001 and sample time 0.1 s to the process output for Example 4. (c) Controller output of the nominal system with white noise of power 0.001 and sample time 0.1 s to the process output for Example 4. This figure is available in colour online at www.apjChemEng.com.

indices are presented in Table 6. Figure 6 shows the closed-loop output responses for a unit-step set-point change occurring at $t = 0$, and an inverse unit-step disturbance at $t = 60$. The proposed method improves the load disturbance and set-point response.

For the robustness study, performance is evaluated by simultaneously inserting a perturbation uncertainty of 5% in all three parameters in the worst direction and finding the actual process according to $G_p = 1.05(-1.05s + 1)e^{-0.525s}\big/(4.5s - 1)(1.9s + 1)(0.475s + 1)$. The simulation results for the model mismatch are given in Table 6, where the proposed method has clear advantage over others. The method by Rao and Chidambaram[2] has a PID with cascaded lead lag filter. Although, it has one additional filter term, the method shows a big overshoot both in the set-point tracking and disturbance rejection.

Example 6. First-order delayed integrating unstable process (FODIUP) with a positive zero. Consider the following FODIUP with a positive zero:

$$G_p = \frac{(-0.5s + 1)e^{-0.2s}}{s(s - 1)} \tag{34}$$

No tuning method able to deal with such an integrating unstable process, which also has a strong positive zero and dead time, has been presented in the literature. The above process has been approximated as containing two unstable poles and dead time according to the following equation:

$$G_p = \frac{100e^{-0.7s}}{(100s - 1)(s - 1)} \tag{35}$$

Although Lee *et al.*[11] have not proposed a design of a PID controller for the FODIUP with a positive zero, such a design is available using the approximated model. For a performance comparison, the proposed method and those of Rao and Chidambaram[2] and Lee *et al.*[11] are considered and the PID setting with performance matrix is listed in Table 7. The λ value is selected to obtain Ms = 12 for each method except that of Rao and Chidambaram.[2] The method of Rao and Chidambaram[2] has been used to obtain the PID setting for the approximated process of $G_p = 100(-0.5s + 1)e^{-0.2s}\big/(100s - 1)(s - 1)$ as mentioned by them. The Rao and Chidambaram[2] method is not able to give any stable closed-loop response and this is probably due to

Table 6. Controller parameters and resulting performance indices for Example 5, $G_p = (-s+1)e^{-0.5s}/(5s-1)(2s+1)(0.5s+1) \approx (-s+1)e^{-0.939s}/(5s-1)(2.07s+1)$.

				Set-point					Disturbance				
				Nominal			5% Mismatch		Nominal			5% Mismatch	
Method	K_c	τ_I	τ_D	ITAE	Overshoot	TV	ITAE	Overshoot	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed $\lambda = 5.302^a$	1.986	24.389	2.399	75.72	1.034	1.696	71.15	1.0	181.4	1.0	4.670	165.5	1.140
Lee *et al.*[11] $\lambda = 6.2537^b$	1.956	34.985	2.488	144.0	1.001	1.421	178.9	1.0	308.2	1.0	4.477	321.7	1.125
Rao and Chidambaram[2] $\lambda = 8.35^c$	1.342	105.83	1.549	3559	2.345	4.168	3584	2.06	2140	2.56	6.79	2433	2.514

Ms = 3.5.

$^a f_R = 1/(21.749s+1)$, $\zeta = 0.71$.

$^b f_R = 1/(32.328s+1)$.

^{c} An additional lead lag filter cascaded with PID controller $f_a = (0.4695s+1)/(0.2406s+1)$ and $f_R = 1/(21.749s+1)$.

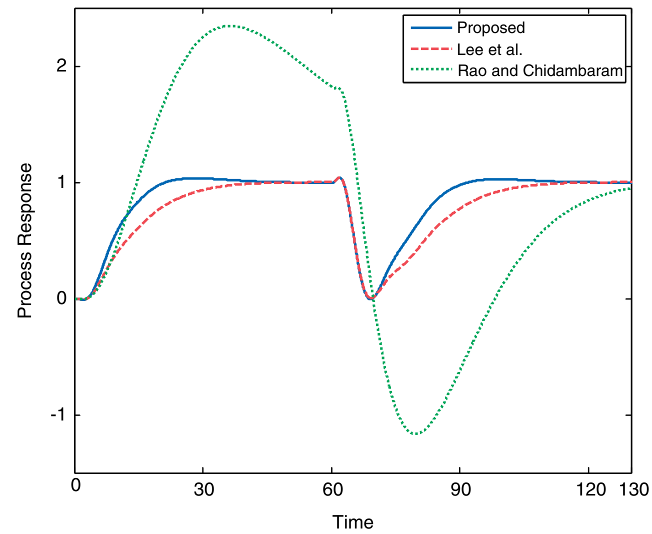

Figure 6. Response of the nominal system for Example 5. This figure is available in colour online at www.apjChemEng.com.

an unstable lag filter. The denominator term of the lead lag filter has negative parameters for any value of the λ for the above process. For this reason, the only PID part of their suggested controller form has been compared for $\lambda = 4$.

A unit step change is added to the set-point at $t = 0$ and an inverse step change of magnitude 0.1 is introduced to the load disturbance at $t = 40$. The simulation results are provided in Fig. 7 and the performance matrix in Table 7. These results demonstrate the advantage of the proposed method in both the set-point and load disturbance responses.

In the case of a 5% error for estimating the process parameters toward the worst case model mismatch, i.e. $G_p = (-0.525s+1)e^{-0.21s}/s(0.95s-1)$, the perturbed system responses and performance matrix are also listed in Table 7. The results demonstrate that the proposed method facilitates robust stability in the presence of process uncertainty over other methods.

DISCUSSION

Performance of proposed controller with variation in ζ

An investigation of the consequences of the variation in the IMC filter parameter ζ is significant because it is the sole parameter determining performance improvement. Consider the example of the FODUP model $G_p = e^{-1.5s}/(s-1)$ studied above. A comparative study by Zhou *et al.*[18] revealed that the PID tuning rule reported in the literature[5–7,15,25] is not valid for large θ/τ ratios and that only the method of Yang *et al.*[12] and Lee *et al.*[11] is viable. Although simulation results of Zhou

Table 7. Controller parameters and resulting performance indices for Example 6, $G_p = (-0.5s + 1)e^{-0.2s}/s(s-1)$.

Method	K_c	τ_I	τ_D	Set-point					Disturbance				
				Nominal			5% Mismatch		Nominal			5% Mismatch	
				ITAE	Overshoot	TV	ITAE	Overshoot	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed $\lambda = 2.135$[a]	0.049	6.318	24.586	53.52	1.119	0.342	58.92	1.115	161.0	1.806	2.096	222.5	2.037
Lee *et al.*[11] $\lambda = 2.144$[b]	0.051	7.840	24.073	65.97	1.076	0.252	84.58	1.087	187.1	1.646	2.005	493.4	1.871
Rao and Chidambaram[2] $\lambda = 4.0$	0.044	10.335	28.466	97.8	1.273	0.8163	260.2	1.396	359.7	1.6432	1.8441	607.8	1.9177

Unable to achieve the lower Ms value and due to that we have selected Ms = 12.0 except in the method of Rao and Chidambaram[2].

[a] $f_R = 1/(154.935s^2 + 6.245s + 1)$, $\zeta = 0.8$.

[b] $f_R = 1/(188.019s^2 + 7.740s + 1)$.

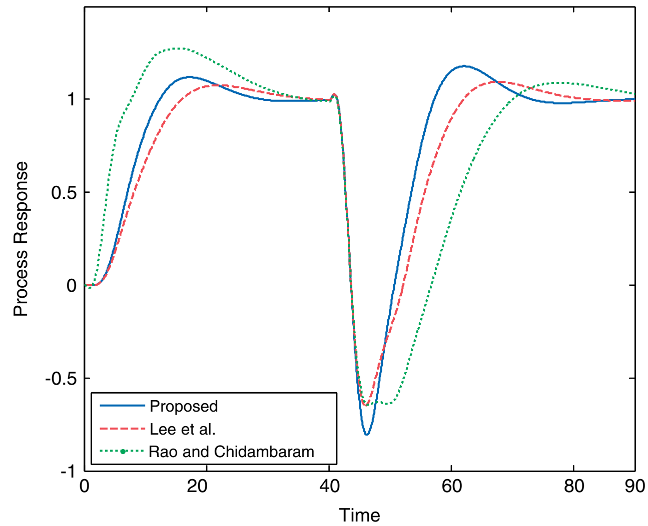

Figure 7. Response of the nominal system for Example 6. This figure is available in colour online at www.apjChemEng.com.

et al.[18] presented different output responses, for the methods of Yang *et al.*[12] and Lee *et al.*[11], this was due to the selection of different λ values between the two methods. When the same Ms value is applied, the resulting performance is identical for the methods of Lee *et al.*[11] and Yang *et al.*[12].

The simulation study was conducted to visualize the effect of ζ for the above FODUP over a wide range of ζ values. The λ value was adjusted for each ζ value to obtain Ms = 29.07, the same as Yang *et al.*[12] The PID parameter setting and performance matrix are listed in Table 8 for several ζ values. Figure 8 shows the output response of the above process for set-point tracking and disturbance rejection. Figure 9 shows the variation of ITAE value with ζ for both the set-point and disturbance rejection. Although the ITAE value is minimized for ζ in the range 0.3–0.4, a ζ value in the range 0.5–0.6 is recommended because it provides an acceptable trade-off between a fast response and reduced oscillation. It appears from Figs 8 and 9 and Table 8 that $\zeta = 1$, which has normally been used by many researchers,[11,12,15] is not the best selection for the IMC filter. The integral action is inadequate with $\zeta = 1$ and this causes the slow response, which is, however, increased with decreasing ζ value. The performance can be improved by increasing the integral action, which can be achieved by utilizing small ζ values in the IMC filter. The desired Ms level and the corresponding optimal ζ value can be varied by altering the normalized time θ/τ.

The simplest filter structure widely used is $f = 1/(\lambda s + 1)^n$, which has the main potential merit of good robustness characteristics with a maximum peak of 1 for $|f|$. However, for the underdamped filter structure of $f = 1/(\lambda^2 s^2 + 2\lambda\zeta s + 1)^n$, the maximum peak for

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Table 8. Performance comparison for different ζ value for $G_p = e^{-1.5s}/(s-1)$.

Tuning methods	λ	K_c	τ_I	τ_D	Set-point			Disturbance		
					TV	ITAE	Overshoot	TV	ITAE	Overshoot
Lee *et al.*[11]/Yang *et al.*[12] $\zeta = 1.0$	6.543	1.064	253.323	0.765	1.013	155.3	0.996	4.701	366.0	1.506
$\zeta = 0.9$	6.065	1.064	218.028	0.763	1.032	114.4	1.002	4.697	283.4	1.507
$\zeta = 0.8$	5.599	1.064	184.936	0.762	1.069	98.81	1.015	4.690	226.6	1.507
$\zeta = 0.7$	5.148	1.064	155.366	0.761	1.133	84.47	1.038	4.692	187.7	1.506
$\zeta = 0.6$	4.719	1.065	129.463	0.759	1.225	69.1	1.084	4.701	163.6	1.506
$\zeta = 0.5$	4.308	1.065	106.724	0.757	1.353	63.49	1.125	4.711	140.9	1.506
$\zeta = 0.4$	3.928	1.065	87.472	0.754	1.554	57.7	1.199	4.731	130.9	1.508
$\zeta = 0.3$	3.581	1.065	71.342	0.751	1.719	61.48	1.240	4.789	117.7	1.514
$\zeta = 0.2$	3.284	1.065	58.455	0.746	2.238	72.0	1.381	4.892	128.6	1.528

Ms = 29.70.

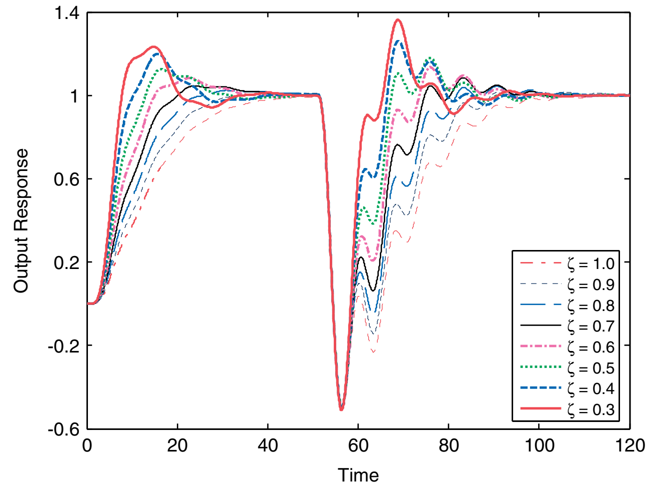

Figure 8. Simulation results for different values of damping coefficient ζ. This figure is available in colour online at www.apjChemEng.com.

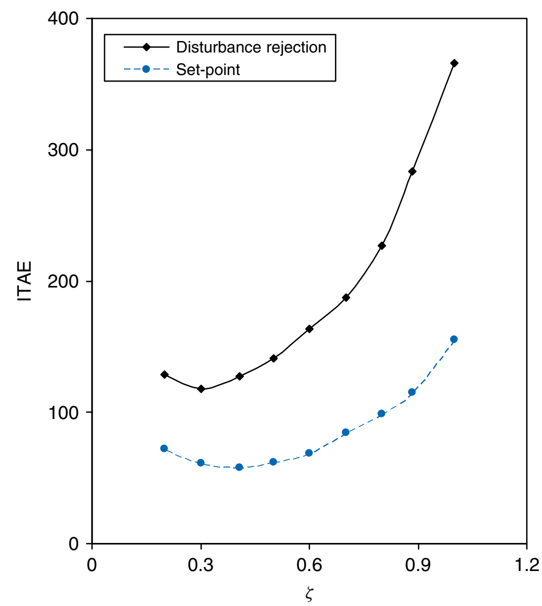

Figure 9. Variation of ITAE with ζ. This figure is available in colour online at www.apjChemEng.com.

$|f|$ is 1 for $\zeta \geq 0.707$. Therefore, in practice, choosing a filter with $\zeta \geq 0.707$ does not introduce any inherent robustness problem in designing the IMC controller.

Analysis of the λ and ζ guidelines for FODUP

This section presents the λ and ζ guidelines at a fixed dead time uncertainty margin. Kharitonov's theorem[21] is used to obtain the uncertainty margin in the process parameter. The dead time uncertainty is selected for the analysis because among all the parameters it shows

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one of the most deleterious effects on the control performance. The analysis can also be performed in a similar manner for other parameter uncertainties such as gain and time constant.

Kharitonov's theorem (Bhattacharyya *et al*.[21]) states that the Hurwitz stability of the real (or complex) interval polynomial family can be guaranteed by the Hurwitz stability of four prescribed critical vertex polynomials in this family. The result is significant since it reduces the checking stability of infinitely many polynomials, and the number of critical vertex polynomials to be checked is independent of the order of the polynomial family.

Every polynomial in the family $\chi(s)$ is Hurwitz stable if and only if the following four extreme polynomials are Hurwitz stable:

$$k_1(s) = \underline{\chi}_0 + \overline{\chi}_1 s + \overline{\chi}_2 s^2 + \underline{\chi}_3 s^3 + \underline{\chi}_4 s^4 + \overline{\chi}_5 s^5 + \overline{\chi}_6 s^6 \ldots \quad (36a)$$

$$k_2(s) = \underline{\chi}_0 + \underline{\chi}_1 s + \overline{\chi}_2 s^2 + \overline{\chi}_3 s^3 + \underline{\chi}_4 s^4 + \underline{\chi}_5 s^5 + \overline{\chi}_6 s^6 \ldots \quad (36b)$$

$$k_3(s) = \overline{\chi}_0 + \underline{\chi}_1 s + \underline{\chi}_2 s^2 + \overline{\chi}_3 s^3 + \overline{\chi}_4 s^4 + \underline{\chi}_5 s^5 + \underline{\chi}_6 s^6 \ldots \quad (36c)$$

$$k_4(s) = \overline{\chi}_0 + \overline{\chi}_1 s + \underline{\chi}_2 s^2 + \underline{\chi}_3 s^3 + \overline{\chi}_4 s^4 + \overline{\chi}_5 s^5 + \underline{\chi}_6 s^6 \ldots \quad (36d)$$

The stability of the above four equations formed from Kharitonov polynomials is to be checked. For fixed values of gain K and time constant τ, a perturbation in time delay θ, i.e. $(\theta - \Delta\theta) \le \theta \le (\theta + \Delta\theta)$, is substituted in the above coefficients and Kharitonov's four equations are checked for stability using the Routh–Hurwitz method. Similar perturbation analysis can also be performed for K, i.e. $(K - \Delta K) \le K \le (K + \Delta K)$ (for fixed τ and θ), and τ, i.e. $(\tau - \Delta\tau) \le \tau \le (\tau + \Delta\tau)$ (for fixed K and θ).

The characteristic equation for the closed-loop system is $1 + G_{OL} = 0$. By substitution of G_{OL} and approximating the dead time term by a high-order Pade approximation, the general form of the characteristic equation $1 + G_{OL} = 0$ for FODUP can be extracted as

$$\chi(s) = \chi_0 + \chi_1 s + \chi_2 s^2 + \chi_3 s^3 + \chi_4 s^4 + \chi_5 s^5 \quad (37)$$

$\underline{\chi}_i \le \chi_i \le \overline{\chi}_i$, (i = 0, 1, 2, 3, 4, 5) where $\underline{\chi}_i$ and $\overline{\chi}_i$ are, respectively, the lower and upper bounds for χ_i.

Consider the control system design of the FODUP by the PID controller. The characteristic equation is given by

$$\frac{K_c K\left(1 + \tau_I s + \tau_I \tau_D s^2\right) e^{-\theta s}}{\tau_I s(\tau s - 1)} + 1 = 0 \quad (38)$$

The time delay term in Eqn (38) can be approximated by using the third-order Pade approximation:

$$e^{-\theta s} = \frac{120 - 60\theta s + 12\theta^2 s^2 - \theta^3 s^3}{120 + 60\theta s + 12\theta^2 s^2 + \theta^3 s^3} \quad (39)$$

Since the process is dead time dominant, the high-order Pade approximation has to be utilized to minimize the approximation error and the coefficient of the characteristic equation is given below:

$$\chi_0 = 120 K_c K \quad (40a)$$

$$\chi_1 = -120\tau_I - 60 K_c K\theta + 120 K_c K \tau_I \quad (40b)$$

$$\chi_2 = 12 K_c K\theta^2 - 60 K_c K \tau_I \theta + 120 K_c K \tau_I \tau_D - 60\tau_I \theta + 120\tau_I \tau \quad (40c)$$

$$\chi_3 = -K_c K\theta^3 + 12 K_c K \tau_I \theta^2 - 60 K_c K \tau_I \tau_D \theta - 12\tau_I \theta^2 + 60\tau_I \tau\theta \quad (40d)$$

$$\chi_4 = -\tau_I \theta^3 + 12\tau_I \tau\theta^2 - K_c K \tau_I \theta^3 + 12 K_c K \tau_I \tau_D \theta^2 \quad (40e)$$

$$\chi_5 = \tau_I \tau\theta^3 - K_c K \tau_I \tau_D \theta^3 \quad (40f)$$

Equation (40) provides the coefficients of the characteristic equation given by Eqn (37) and the uncertainty in the process parameters can be checked for stability of all four Kharitonov polynomials in Eqn (36). On the basis of Kharitonov's theorem described above, the dead time uncertainty margin is fixed to obtain the λ guideline. The closed-loop time constant λ is a user-defined, tuning parameter in the proposed tuning rule and is directly related to the performance and robustness of the control system. For this reason, it is important to have a λ guideline which can give a fast and robust response for a given θ/τ ratio.

In the proposed study, the ζ value is selected such that it improves the integral action and thus the performance. Figure 10 shows the plot of λ/τ verses θ/τ for FODUP over a wide range of ζ values for 5% dead time uncertainty margin. The λ value is obtained for the 5% dead time uncertainty at different ζ values by using Kharitonov's theorem, as shown above. Similarly, the λ/τ verses θ/τ plot for 10% dead time uncertainty margin is presented in Fig. 11. To visualize the effect in terms of performance improvement, Fig. 12 shows the ITAE values according to the θ/τ ratio for 5% dead time uncertainty margin. The same trend is evident at different uncertainty margins. Figures 10 and 12 show that, for a fixed θ/τ ratio, λ decreases with decreasing ζ, which improves ITAE for a fixed θ/τ ratio. As seen from the figures, as the value θ/τ increases, the λ/τ value to secure stability grows rapidly. It is important to mention that for a large θ/τ, it is not possible to obtain a stable response even by using a high λ/τ value.

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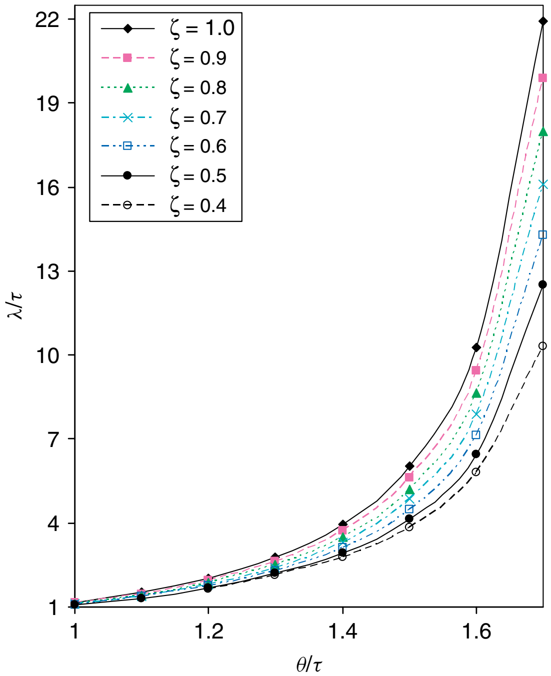

Figure 10. λ Guidelines for 5% θ uncertainty for the FODUP. This figure is available in colour online at www.apjChemEng.com.

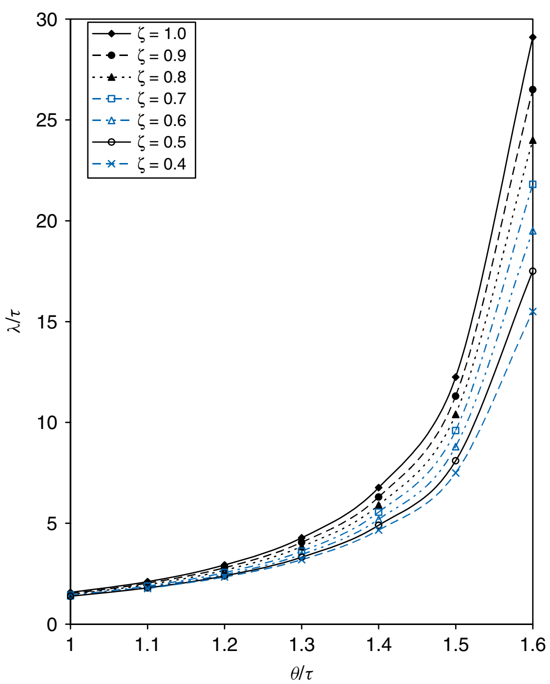

Figure 11. λ Guidelines for 10% θ uncertainty for the FODUP. This figure is available in colour online at www.apjChemEng.com.

For the FODUP, the suggested IMC filter in the proposed study is $f = (\beta s + 1)\Big/\left(\lambda^2 s^2 + 2\lambda\zeta s + 1\right)$ and the resulting output response for the disturbance

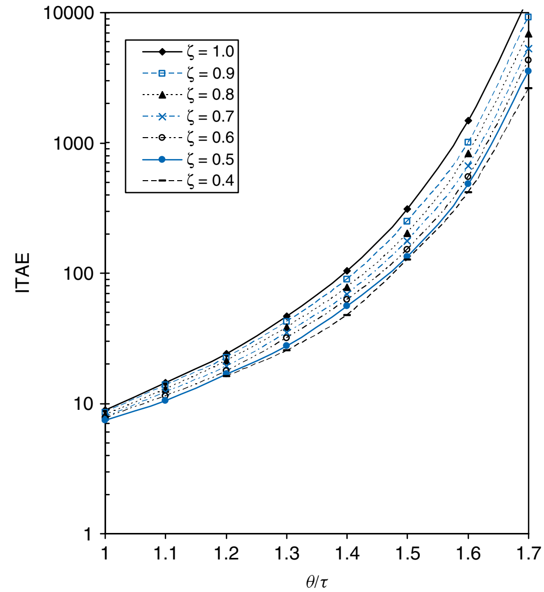

Figure 12. ITAE variation with θ/τ ratio at several ζ values for the FODUP. This figure is available in colour online at www.apjChemEng.com.

rejection is given by

$$\frac{C}{d} = \frac{K(\lambda^2 s^2 + 2\lambda\zeta s + 1)e^{-\theta s} - K(\beta s + 1)e^{-2\theta s}}{(\tau s - 1)(\lambda^2 s^2 + 2\lambda\zeta s + 1)} \tag{41}$$

Since β is designed so that the denominator in Eqn (41) contains $(\tau s - 1)$, the speed of the output response is dependent only on $(\lambda^2 s^2 + 2\lambda\zeta s + 1)$. The significant observation as perceived earlier (Figs 10 and 11) is that the required λ value for the same robustness level decreases with decreasing ζ. Values of ζ in the range 0.4–0.8 are often suitable for a desired control system response in which the controlled variable reaches the set-point faster and in a stable manner than that with $\zeta = 1$.

CONCLUSIONS

The present study is focused on unstable processes comprising negative and positive zeros with dead time. The following key conclusions are made:

1. The IMC filter is modified from a critically damped to an underdamped structure for the dead time dominant unstable process. The underdamped filter provides the desired level of integral action and thereby improves the performance at the same robustness level.
2. The inverse response time constant in unstable processes with a positive zero can be approximated as a time delay for the design of the PID controller which simplifies the design for improved performance.

3. For unstable processes with a strong lead-time constant, an overdamped filter is suggested because it significantly reduces both the overshoot and undershoot. In this case, a set-point filter is not required for the servo response, whereas it is needed to eliminate the overshoot for $\zeta = 1$.
4. The simulation study conducted to show the effect of ζ variation on the performance at a fixed Ms value clearly reveals that a critically damped filter is not the best option for PID controller design.
5. By using Kharitonov's theorem, closed-loop time constant λ guidelines are suggested for the 5% and 10% dead time uncertainty margins at several ζ values.
6. The simulation studies conducted for several process classes clearly demonstrate the clear advantage of the proposed method over others previously reported.

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